Eur. Phys. J. C 50, 935–948 (2007)
DOI 10.1140/epjc/s10052-007-0264-0

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article – Theoretical Physics

Different kind of textures of Yukawa coupling matrices in the two Higgs doublet model type III

A. Cárcamo[1,2,a], R. Martinez[2,b], J.-A. Rodríguez[2,c]

[1] Scuola Normale Superiore, Piazza dei Cavalieri 7, 56126 Pisa, Italy
[2] Universidad Nacional de Colombia, Departamento de Física, Bogotá, Colombia

Received: 25 November 2006 / Revised version: 8 February 2007 /
Published online: 20 March 2007 –

Abstract. The quark mass matrices ansätze proposed by Fritzsch, Du–Xing and Fukuyama–Nishiura in the framework of the general two Higgs doublet model are studied. The corresponding Yukawa matrices in the flavor basis in the various cases considered are discussed. The corresponding Cabibbo–Kobayashi–Maskawa matrix elements are computed in all cases and compared with their experimental values. The complex phases of the ansätze are taken into account, and the CP violating phase δ is computed. Finally, in order to observe the influence of the various kinds of texture of the Yukawa coupling matrices considered, some issues in the phenomenology of the two body decays of the top quark, the lightest Higgs boson and the charged Higgs boson are discussed.

1 Introduction

Despite all its success, the standard model (SM) of the electroweak interactions based on the $\mathrm{SU}(3)_C \otimes \mathrm{SU}(2)_\mathrm{L} \otimes \mathrm{U}(1)_Y$ gauge symmetry has many unexplained features. Most of them are linked to the fermionic sector, such as the origin of the fermion masses and the mixing angles [1–11]. These features lead one to consider the SM not to be the most fundamental theory of the basic interactions. It should be considered as an effective theory that remains valid up to some energy scale of the order of TeV, and eventually it will be replaced by a more fundamental theory. In the framework of the SM, the values of the Yukawa couplings are parameterized in a phenomenological way. The quark masses and mixing matrix are described by 10 free parameters: six quark masses, three flavor mixing angles and one CP violating phase. The form of the Yukawa couplings is not well understood, and neither are their origin nor the underlying principles; this is known as the flavor problem. Attempts to compute these 10 phenomenological parameters have been made within the framework of the extensions of the SM, including grand unification theories, supersymmetric theories and superstring theories [12–15]. There are two basic approaches to study the patterns of Yukawa couplings: one way is to adopt the unification hypothesis of matter multiplets, and the second one is to assume a specific form for the Yukawa couplings, called texture [16, 17]. The two approaches are strongly related, because the mass matrices gotten using textures could be incorporated into grand unified theories [12, 13]. The use of quark mass matrix textures is motivated by the observed large hierarchies of the quark masses and the Cabibbo–Kobayashi–Maskawa (CKM) matrix elements [22–25, 27–29]. The phenomenological quark mass matrices have been discussed from different points of view. For example, quark mass matrix ansätze have been used in an analogous way in the leptonic sector, trying to get the lepton mixing matrix in order to explain the neutrino anomalies [18–21, 28–34]. Also, new texture ansätze have been proposed motivated by more precise results on the elements of the quark mixing matrix [1–4, 22–25, 35]. Quark and lepton mass matrices have also been discussed in the context of SO(10) grand unification theories [13]. The understanding of the discrete flavor symmetries hidden in such textures may be useful in the knowledge of the underlying dynamics responsible for quark mass generation and CP violation.

One possible simple extension of the SM is to add a new Higgs doublet, and this is called the two Higgs doublet model (2HDM). This extension has the following direct consequences: it increases the scalar spectrum, and it gives a more generic pattern of the flavor changing neutral currents (FCNC). FCNC at tree level can be considered a problem that was solved in the earlier versions of the two Higgs doublet model (2HDM type I and II) by imposing a discrete symmetry that enforces the restriction that each fermion be coupled at most to one Higgs doublet [36]. But if the discrete symmetry is not imposed, then FCNC at tree level remains; it is the so-called two Higgs doublet model type III. The 2HDM type III and its phenomenology have been extensively studied in the literature [37–41]. In 2HDM-III, for each quark type, up or down, there are

[a] e-mail: antonio.carcamo@sns.it, aecarcamoh@unal.edu.co
[b] e-mail: remartinezm@unal.edu.co
[c] e-mail: jarodriguezl@unal.edu.co

two Yukawa couplings. One of the Yukawa couplings is for generating the quark masses, and the other one produces the flavor changing couplings at tree level. But in any case both Higgs doublets have the same quantum numbers; then if the one ansatz is assumed for the Yukawa coupling that generates the quark masses, it is valid to associate the same structure for the other Yukawa coupling, which is generating the flavor changing couplings. This is the main hypothesis of this work.

In this manuscript, different kind of textures of the Yukawa coupling matrices are discussed in the framework of the 2HDM type III model. In Sect. 2, the principal features of 2HDM type III are reviewed; next, in Sect. 3, the different kind of texture ansätze are introduced. First of all, the Fritzsch ansatz and the Du–Xing ansatz are reviewed. Then, the Fritzsch and Du–Xing ansätze are combined, searching for better agreement with the experimental results. After that, two different assignments of the Du–Xing ansatz are combined. A brief review of the Fukuyama–Nishiura mass matrix ansatz is presented, and finally a new ansatz is proposed. The new ansatz presents the feature that the top quark does not have any mixing, and therefore it does not show any flavor changing neutral process at tree level. The corresponding Yukawa coupling matrices in the mass basis in the framework of 2HDM type III are gotten for all the ansätze discussed. This means that the intensity of the couplings that lead to FCNC processes are obtained. Also, the numerical values of the corresponding CKM matrix elements are obtained. In Sect. 4, some phenomenological aspects of the two body decays of the top quark, the lightest Higgs boson and the charged Higgs boson are discussed. Finally, in Sect. 5 our conclusions are presented.

2 Yukawa interaction Lagrangian for the two Higgs doublet model

In the most general form of 2HDM, the Lagrangian for the Yukawa interaction in the quark sector is given by [42, 43]

$$\begin{aligned} -L_{\mathrm{Y}} = & \bar{q}_{\mathrm{L}}^0 \eta^{U,0} \tilde{\phi}_1 U_{\mathrm{R}}^0 + \bar{q}_{\mathrm{L}}^0 \eta^{D,0} \phi_1 D_{\mathrm{R}}^0 \\ & + \bar{q}_{\mathrm{L}}^0 \xi^{U,0} \tilde{\phi}_2 U_{\mathrm{R}}^0 + \bar{q}_{\mathrm{L}}^0 \xi^{D,0} \phi_2 D_{\mathrm{R}}^0 + \mathrm{h.c.}, \end{aligned} \tag{1}$$

where $\eta^{f,0}$ and $\xi^{f,0}$ are the Yukawa interaction matrices, and where $\tilde{\phi} = \mathrm{i}\tau_2 \phi^*$, $f^0 = U^0, D^0$; $q_{\mathrm{L}}^0 = (U^0, D^0)_{\mathrm{L}}$ are the quark doublet states in the interaction basis, and $U^0 = (\mathrm{u}^0, \mathrm{c}^0, \mathrm{t}^0)$, $D^0 = (\mathrm{d}^0, \mathrm{s}^0, \mathrm{b}^0)$ are the quark interaction eigenstates. The two Higgs boson fields, after spontaneous symmetry breaking, have the following form:

$$\phi_k = \begin{pmatrix} \phi_k^+ \\ \frac{1}{\sqrt{2}}\left(v_k + \phi_k^0 + \mathrm{i}\lambda_k^0\right) \end{pmatrix} \quad \text{with} \quad k = 1, 2, \tag{2}$$

where v_1 and v_2 are the vacuum expectation values of the two Higgs fields ϕ_1 and ϕ_2, respectively. There is no extra discrete symmetry for the Higgs fields. This means that they can mix, because they have the same quantum numbers. Furthermore, both Higgs doublets can simultaneously give mass to the down- and up-type quarks. However, the Higgs fields can be rotated to a new base, where

$$\begin{pmatrix} \phi_1' \\ \phi_2' \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}, \tag{3}$$

in which case $\tan\beta = v_2/v_1$ and the vacuum expectation values are $\langle \phi_1' \rangle = v = \sqrt{v_1^2 + v_2^2}$ and $\langle \phi_2' \rangle = 0$. In this article, we do not consider spontaneous CP symmetry breaking in the Higgs sector. In the new base mentioned, the Yukawa Lagrangian can be written in exactly the same way as in (1), but the matrices $\eta^{f,0}$, $\xi^{f,0}$ will be rotated to a new base:

$$\begin{pmatrix} \eta'^{f,0} \\ \xi'^{f,0} \end{pmatrix} = \begin{pmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \eta^{f,0} \\ \xi^{f,0} \end{pmatrix}. \tag{4}$$

In general, the quantum system will be in the primed base, where only one vacuum expectation value is different from zero [37–41]. We are going to use this base for the Yukawa sector, but from now on we will omit the prime symbol. In this new base, it can be shown that in the scalar sector there are five scalar bosons,

$$\phi^{\pm} = H^{\pm}, \quad \lambda_2^0 = A^0, \quad \phi_1^0, \quad \phi_2^0, \tag{5}$$

where ϕ_1^0, ϕ_2^0 are rotated; in terms of mass eigenstates they are

$$\begin{pmatrix} h^0 \\ H^0 \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \phi_1^0 \\ \phi_2^0 \end{pmatrix}, \tag{6}$$

where the parameter α can be taken in the range $-\frac{\pi}{2} \leqslant \alpha \leqslant \frac{\pi}{2}$ [42, 43]. Thus, from the scalar potential five Higgs boson mass eigenstates arise; they are H^0, h^0, $H^{\pm}$ and A^0.

From the terms of the Lagrangian of (1), we see that the mass matrices for the fermions are given by

$$M^f = \frac{v}{\sqrt{2}} \eta^{f,0}. \tag{7}$$

Also, the Hermitian mass matrix M is diagonalized by a rotation matrix, according to

$$V^{\dagger} M V = O^{\mathrm{T}} P^{\dagger} M P O = O^{\mathrm{T}} \widetilde{M} O = \begin{pmatrix} \pm m_1 & 0 & 0 \\ 0 & \mp m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}, \tag{8}$$

where $\widetilde{M} = P^{\dagger} M P$ is a real symmetric mass matrix, O is an orthogonal matrix, $P = \mathrm{diag}(1, \mathrm{e}^{-\mathrm{i}\psi}, \mathrm{e}^{-\mathrm{i}(\psi+\theta)})$, and m_1, m_2 and m_3 correspond to the fermion masses. The upper signs are associated to the Fritzsch and Du–Xing ansätze, while the lower ones are used for the Fukuyama–Nishiura ansatz (see Sect. 3).

The fermion mass eigenstates are related to the interaction eigenstates by bi-unitary transformations [37–41],

$$\begin{aligned} U_{\mathrm{L}}^0 &= V_{\mathrm{L}}^U U_{\mathrm{L}}, & U_{\mathrm{R}}^0 &= V_{\mathrm{R}}^U U_{\mathrm{R}}, \\ D_{\mathrm{L}}^0 &= V_{\mathrm{L}}^D D_{\mathrm{L}}, & D_{\mathrm{R}}^0 &= V_{\mathrm{R}}^D D_{\mathrm{R}}, \end{aligned} \tag{9}$$

and therefore the CKM matrix will be defined by

$$K = \left(V_{\rm L}^U\right)^\dagger V_{\rm L}^D = O_U^{\rm T} P_{UD} O_D \,, \\ P_{UD} = P_U^\dagger P_D = {\rm diag}(1, {\rm e}^{{\rm i}\sigma}, {\rm e}^{{\rm i}\tau}) \,. \qquad (10)$$

Here $\sigma = \psi_U - \psi_D$ and $\tau = \psi_U - \psi_D + \theta_U - \theta_D$. $\psi_{U,D}$ and $\theta_{U,D}$ are the phases of the mass matrices for the up- and down-type quarks, respectively. Further, when the transition to the mass eigenstates is performed in the Yukawa Lagrangian (1), the following relations, corresponding to the "rotated" coupling matrices, are gotten [37–41]:

$$M_U^{\rm diag} = \frac{v}{\sqrt{2}} V_{\rm L}^{U\dagger} \eta^{U,0} V_{\rm R}^U \,, \quad M_D^{\rm diag} = \frac{v}{\sqrt{2}} V_{\rm L}^{D\dagger} \eta^{D,0} V_{\rm R}^D \,, \\ \xi^U = V_{\rm L}^{U\dagger} \xi^{U,0} V_{\rm R}^U \,, \qquad \xi^D = V_{\rm L}^{D\dagger} \xi^{D,0} V_{\rm R}^D \,. \qquad (11)$$

Then eliminating $\eta^{f,0}$ and $\xi^{f,0}$ using (11), the Yukawa Lagrangian corresponding to the quark sector for the two Higgs doublet model type III takes the following form [37–43]:

$$\begin{aligned} -L_{\rm Y} = & \overline{U} M_U^{\rm diag} U + \frac{1}{v} \overline{U} M_U^{\rm diag} U \left(\cos\alpha H^0 - \sin\alpha h^0\right) \\ & + \frac{1}{\sqrt{2}} \overline{U} \left(\xi^U P_{\rm R} + (\xi^U)^\dagger P_{\rm L}\right) U \left(\sin\alpha H^0 + \cos\alpha h^0\right) \\ & + \overline{D} M_D^{\rm diag} D + \frac{1}{v} \overline{D} M_D^{\rm diag} D \left(\cos\alpha H^0 - \sin\alpha h^0\right) \\ & + \frac{1}{\sqrt{2}} \overline{D} \left(\xi^D P_{\rm R} + (\xi^D)^\dagger P_{\rm L}\right) D \left(\sin\alpha H^0 + \cos\alpha h^0\right) \\ & + \frac{\rm i}{v} \overline{U} M_U^{\rm diag} \gamma_5 U G_Z^0 - \frac{\rm i}{v} \overline{D} M_D^{\rm diag} \gamma_5 D G_Z^0 \\ & - \frac{\rm i}{\sqrt{2}} \overline{U} \left(\xi^U P_{\rm R} - (\xi^U)^\dagger P_{\rm L}\right) U A^0 \\ & + \frac{\rm i}{\sqrt{2}} \overline{D} \left(\xi^D P_{\rm R} - (\xi^D)^\dagger P_{\rm L}\right) D A^0 \\ & + \frac{\sqrt{2}}{v} \overline{U} \left(K M_D^{\rm diag} P_{\rm R} - M_U^{\rm diag} K P_{\rm L}\right) D G_W^+ \\ & - \frac{\sqrt{2}}{v} \overline{D} \left(K^\dagger M_U^{\rm diag} P_{\rm R} - M_D^{\rm diag} K^\dagger P_{\rm L}\right) U G_W^- \\ & + \overline{U} \left(K \xi^D P_{\rm R} - (\xi^U)^\dagger K P_{\rm L}\right) D H^+ \\ & - \overline{D} \left(K^\dagger \xi^U P_{\rm R} - (\xi^D)^\dagger K^\dagger P_{\rm L}\right) U H^- \,. \end{aligned} \qquad (12)$$

Notice that if ξ^U and ξ^D vanish from this Yukawa Lagrangian, 2HDM type II is reproduced. The new terms lead to flavor changing processes at tree level.

3 Different kind of ansätze

In this section different mass matrices ansätze are presented. The Yukawa coupling matrices in the mass basis are obtained, assuming that the Yukawa coupling matrices have the same structure that the mass matrices. In the framework of 2HDM type III the flavor changing couplings at tree level will depend on the ansatz parameters. The Fritzsch ansatz is the first one reviewed, because it is the simplest one, and it was the motivation for the widely known and used Cheng–Sher ansatz in the 2HDM-III phenomenology. Next, the Du–Xing ansatz is discussed and it is followed by combinations of the Fritzsch and the Du–Xing ansätze,where we look for the best fit of the CKM elements. Furthermore, two different assignments of the Du–Xing ansatz are combined; the Fukuyama–Nishiura ansatz is explored, and a new type of ansatz, which controls the FCNC processes in the top quark sector, is proposed. It is worth to mention that phases are explicitly involved, since the mass matrices are hermitian, and therefore they can always be written as $M = P\widetilde{M}P^\dagger$, where $\widetilde{M}$ is a real and symmetric matrix, and where P is a diagonal matrix that contains the phases [23–25].

3.1 Fritzsch ansatz (FA)

For the three family case, a mass matrix ansatz proposed by Fritzsch has been widely discussed [1–4, 22, 28, 29]; it is

$$\begin{aligned} M &= \begin{pmatrix} 0 & D & 0 \\ D^* & 0 & B \\ 0 & B^* & A \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & {\rm e}^{-{\rm i}\psi} & 0 \\ 0 & 0 & {\rm e}^{-{\rm i}(\psi+\theta)} \end{pmatrix} \begin{pmatrix} 0 & |D| & 0 \\ |D| & 0 & |B| \\ 0 & |B| & A \end{pmatrix} \\ &\quad \times \begin{pmatrix} 1 & 0 & 0 \\ 0 & {\rm e}^{{\rm i}\psi} & 0 \\ 0 & 0 & {\rm e}^{{\rm i}(\psi+\theta)} \end{pmatrix} \\ &= P\widetilde{M}P^\dagger . \end{aligned} \qquad (13)$$

Motivated by the Fritzsch texture mass matrix, Cheng and Sher have proposed an ansatz for the Yukawa flavor changing coupling matrices in the interaction basis according to [22, 28, 29]

$$\begin{aligned} \xi^{f,0} &= \frac{\sqrt{2}}{v} \begin{pmatrix} 0 & dD & 0 \\ dD^* & 0 & bB \\ 0 & bB^* & aA \end{pmatrix} \\ &= \frac{\sqrt{2}}{v} \begin{pmatrix} 1 & 0 & 0 \\ 0 & {\rm e}^{-{\rm i}\psi} & 0 \\ 0 & 0 & {\rm e}^{-{\rm i}(\psi+\theta)} \end{pmatrix} \\ &\quad \times \begin{pmatrix} 0 & d|D| & 0 \\ d|D| & 0 & b|B| \\ 0 & b|B| & aA \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & {\rm e}^{{\rm i}\psi} & 0 \\ 0 & 0 & {\rm e}^{{\rm i}(\psi+\theta)} \end{pmatrix} \\ &= P\tilde{\xi}^{f,0} P^\dagger , \end{aligned} \qquad (14)$$

where $f = U, D$, the coefficients a, b, d are of the order of one, and the parameters A, $|B|$ and $|D|$ are given by

$$A = m_3 - m_2 + m_1 \,, \quad |D| = \left(\frac{m_1 m_2 m_3}{m_3 - m_2 + m_1}\right)^{1/2}, \\ |B| = \left(m_1 m_2 + m_3 m_2 - m_1 m_3 - \frac{m_1 m_2 m_3}{m_3 + m_1 - m_2}\right)^{1/2}. \qquad (15)$$

Table 1. The CKM matrix elements and CP violating phase δ for the different ansätze considered

	FA	DXA	DXFA	FDXA	$\widetilde{\mathrm{X}}$A	FNA	NTA
$\lvert K_{ud}\rvert$	1.00189	1.00189	1.00189	1.00189	1.00189	0.97467	1.00051
$\lvert K_{us}\rvert$	0.22720	0.22720	0.22720	0.22720	0.22720	0.22361	0.22720
$\lvert K_{ub}\rvert$	0.00797	0.00090	0.00734	0.00359	0.00181	0.00308	0.00190
$\lvert K_{cd}\rvert$	0.22710	0.22721	0.22710	0.22710	0.22718	0.22341	0.22720
$\lvert K_{cs}\rvert$	1.00144	1.00194	1.00146	1.00144	1.00178	0.97381	1.00051
$\lvert K_{cb}\rvert$	0.15512	0.01746	0.14277	0.06986	0.03562	0.04221	0.03197
$\lvert K_{td}\rvert$	0.03559	0.00401	0.03275	0.01603	0.00817	0.01002	0.00735
$\lvert K_{ts}\rvert$	0.15512	0.01746	0.14277	0.06986	0.03562	0.04112	0.03203
$\lvert K_{tb}\rvert$	0.99955	1.00004	0.99956	0.99955	0.99988	0.99910	1
δ	86.61°	86.66°	86.62°	86.66°	82.18°	−87.55°	80.68°

But taking into account the mass hierarchy $m_1 \ll m_2 \ll m_3$, the expressions (15) reduce to

$$A \approx m_3\,, \quad |B| \approx \sqrt{m_2 m_3}\,, \quad |D| \approx \sqrt{m_1 m_2}\,. \tag{16}$$

The rotation matrix that diagonalizes $\widetilde{M}$ is given by

$$O \simeq \begin{pmatrix} 1 & -\sqrt{\frac{m_1}{m_2}} & 0 \\ \sqrt{\frac{m_1}{m_2}} & 1 & \sqrt{\frac{m_2}{m_3}} \\ -\sqrt{\frac{m_1}{m_3}} & -\sqrt{\frac{m_2}{m_3}} & 1 \end{pmatrix}. \tag{17}$$

Then using the mass matrix (13) for the up-type and down-type quarks and (10) and (17), the following expressions for the CKM matrix elements are gotten:

$$\begin{aligned}
K_{ud} &\simeq 1 + e^{i\sigma}\sqrt{\frac{m_u m_d}{m_c m_s}} + e^{i\tau}\sqrt{\frac{m_u m_d}{m_t m_b}}\,,\\
K_{us} &\simeq e^{i\sigma}\sqrt{\frac{m_u}{m_c}} - \sqrt{\frac{m_d}{m_s}} + e^{i\tau}\sqrt{\frac{m_u m_s}{m_t m_b}}\,,\\
K_{ub} &\simeq \sqrt{\frac{m_u}{m_c}}\left(e^{i\sigma}\sqrt{\frac{m_s}{m_b}} - e^{i\tau}\sqrt{\frac{m_c}{m_t}}\right),\\
K_{cd} &\simeq e^{i\sigma}\sqrt{\frac{m_d}{m_s}} - \sqrt{\frac{m_u}{m_c}} + e^{i\tau}\sqrt{\frac{m_c m_d}{m_t m_b}}\,,\\
K_{cs} &\simeq e^{i\sigma} + \sqrt{\frac{m_u m_d}{m_c m_s}} + e^{i\tau}\sqrt{\frac{m_c m_s}{m_t m_b}}\,,\\
K_{cb} &\simeq e^{i\sigma}\sqrt{\frac{m_s}{m_b}} - e^{i\tau}\sqrt{\frac{m_c}{m_t}}\,,\\
K_{td} &\simeq \sqrt{\frac{m_d}{m_s}}\left(e^{i\sigma}\sqrt{\frac{m_c}{m_t}} - e^{i\tau}\sqrt{\frac{m_s}{m_b}}\right),\\
K_{ts} &\simeq e^{i\sigma}\sqrt{\frac{m_c}{m_t}} - e^{i\tau}\sqrt{\frac{m_s}{m_b}}\,,\\
K_{tb} &\simeq e^{i\tau} + e^{i\sigma}\sqrt{\frac{m_c m_s}{m_t m_b}}\,.
\end{aligned} \tag{18}$$

In order to get the best agreement of the $|K_{us}|$ and $|K_{cd}|$ elements with their experimental values, we use $\sigma = 81.57°$ and $\tau = -11.78°$. Hence $\psi_U = \psi_D + 81.57°$ and $\theta_U = \theta_D - 93.35°$. The numerical values shown in Table 1 for the CKM matrix elements are obtained using the mass values listed in Appendix B. From these results, considerable disagreement of the $|K_{ub}|$, $|K_{cb}|$, $|K_{td}|$ and $|K_{ts}|$ elements with their experimental values is noticed. On the other hand, the CP violating phase δ for the Fritzsch ansatz is obtained as $\delta = 86.61°$ by using [23–25]

$$\sin\delta = \frac{\left(1 - |K_{ub}|^2\right) J}{|K_{ud} K_{us} K_{ub} K_{cb} K_{tb}|}\,, \tag{19}$$

where J is the Jarlskog invariant given by [23–26]:

$$J = \mathrm{Im}(K_{us} K_{cb} K^*_{ub} K^*_{cs})\,. \tag{20}$$

This CP phase δ presents a discrepancy of about 30.54% with respect to the experimental value $\delta = 60.16° \pm 14°$.

On the other hand, the Yukawa coupling matrices in the mass basis are given by [22]

$$\tilde{\xi}^{U,D} = O^{\mathrm{T}}_{U,D} \tilde{\xi}^{U,D,0} O_{U,D}\,, \tag{21}$$

where $\tilde{\xi}^{U,D,0} = P^\dagger \xi^{U,D,0} P$. Therefore, taking into account the mass hierarchy and using the expression $\xi^{U,D} = P O^{\mathrm{T}}_{U,D} \tilde{\xi}^{U,D,0} O_{U,D} P^\dagger$, the Yukawa coupling matrices in the mass basis are given by

$$\xi^U \simeq \frac{\sqrt{2}}{v}\begin{pmatrix} \chi^{(U)}_{11} m_u & \chi^{(U)}_{12}\sqrt{m_u m_c} & \chi^{(U)}_{13}\sqrt{m_u m_t} \\ \chi^{(U)}_{21}\sqrt{m_u m_c} & \chi^{(U)}_{22} m_c & \chi^{(U)}_{23}\sqrt{m_c m_t} \\ \chi^{(U)}_{31}\sqrt{m_u m_t} & \chi^{(U)}_{32}\sqrt{m_c m_t} & \chi^{(U)}_{33} m_t \end{pmatrix}, \tag{22}$$

$$\xi^D \simeq \frac{\sqrt{2}}{v}\begin{pmatrix} \chi^{(D)}_{11} m_d & \chi^{(D)}_{12}\sqrt{m_d m_s} & \chi^{(D)}_{13}\sqrt{m_d m_b} \\ \chi^{(D)}_{21}\sqrt{m_d m_s} & \chi^{(D)}_{22} m_s & \chi^{(D)}_{23}\sqrt{m_s m_b} \\ \chi^{(D)}_{31}\sqrt{m_d m_b} & \chi^{(D)}_{32}\sqrt{m_s m_b} & \chi^{(D)}_{33} m_b \end{pmatrix}, \tag{23}$$

where

$$\chi_{11}^{(U,D)} = 2d^{(U,D)} - 2b^{(U,D)} + a^{(U,D)},$$
$$\chi_{12}^{(U,D)} = \left(\chi_{21}^{(U,D)}\right)^* = (a^{(U,D)} + d^{(U,D)} - 2b^{(U,D)})\mathrm{e}^{\mathrm{i}\psi_{U,D}},$$
$$\chi_{22}^{(U,D)} = a^{(U,D)} - 2b^{(U,D)}, \quad (24)$$
$$\chi_{13}^{(U,D)} = \left(\chi_{31}^{(U,D)}\right)^* = (b^{(U,D)} - a^{(U,D)})\mathrm{e}^{\mathrm{i}(\psi_{U,D}+\theta_{U,D})},$$
$$\chi_{23}^{(U,D)} = \left(\chi_{32}^{(U,D)}\right)^* = (b^{(U,D)} - a^{(U,D)})\mathrm{e}^{\mathrm{i}\theta_{U,D}},$$
$$\chi_{33}^{(U,D)} = a^{(U,D)}. \quad (25)$$

Equations (22) and (35) reproduce the expression obtained by Cheng and Sher [22] if the complex phases vanish. Further, we can see that the hierarchical structure of the Yukawa coupling matrices in the mass basis is dominated by the fermion masses and the $(\xi^{U,D})_{ij}$ elements are proportional to $\sqrt{m_i m_j}$, which is the usual Cheng–Sher ansatz [22].

3.2 Du–Xing ansatz (DXA)

In order to accommodate all the current data of quark masses and mixing angles in the framework of zero textures, the following ansatz for the mass matrix was suggested [1–4, 22, 28, 29]:

$$M = \begin{pmatrix} 0 & D & 0 \\ D^* & C & B \\ 0 & B^* & A \end{pmatrix} = P\widetilde{M}P^\dagger \text{ with } |A| \gg |B|\,,\ |C| \gg |D|\,, \quad (26)$$

where $A \simeq m_3$, $|B| \simeq |C| \simeq m_2$, and $|D| \simeq \sqrt{m_1 m_2}$. The rotation matrix for this case is [22]

$$O \simeq \begin{pmatrix} 1 & \sqrt{\frac{m_1}{m_2}} & 0 \\ -\sqrt{\frac{m_1}{m_2}} & 1 & \frac{m_2}{m_3} \\ \frac{\sqrt{m_1 m_2}}{m_3} & -\frac{m_2}{m_3} & 1 \end{pmatrix}. \quad (27)$$

The CKM matrix elements obtained from the mass matrix ansatz given in (26) are [28, 29]

$$K_{\mathrm{ud}} \simeq 1 + \mathrm{e}^{\mathrm{i}\sigma}\sqrt{\frac{m_\mathrm{u} m_\mathrm{d}}{m_\mathrm{c} m_\mathrm{s}}} + \mathrm{e}^{\mathrm{i}\tau}\frac{\sqrt{m_\mathrm{u} m_\mathrm{c} m_\mathrm{d} m_\mathrm{s}}}{m_\mathrm{t} m_\mathrm{b}}\,,$$
$$K_{\mathrm{us}} \simeq \sqrt{\frac{m_\mathrm{d}}{m_\mathrm{s}}} - \mathrm{e}^{\mathrm{i}\sigma}\sqrt{\frac{m_\mathrm{u}}{m_\mathrm{c}}} - \mathrm{e}^{\mathrm{i}\tau}\frac{m_\mathrm{s}\sqrt{m_\mathrm{u} m_\mathrm{c}}}{m_\mathrm{t} m_\mathrm{b}}\,,$$
$$K_{\mathrm{ub}} \simeq \sqrt{\frac{m_\mathrm{u}}{m_\mathrm{c}}}\left(\frac{m_\mathrm{c}}{m_\mathrm{t}}\mathrm{e}^{\mathrm{i}\tau} - \frac{m_\mathrm{s}}{m_\mathrm{b}}\mathrm{e}^{\mathrm{i}\sigma}\right),$$
$$K_{\mathrm{cd}} \simeq \sqrt{\frac{m_\mathrm{u}}{m_\mathrm{c}}} - \mathrm{e}^{\mathrm{i}\sigma}\sqrt{\frac{m_\mathrm{d}}{m_\mathrm{s}}} - \mathrm{e}^{\mathrm{i}\tau}\frac{m_\mathrm{c}\sqrt{m_\mathrm{d} m_\mathrm{s}}}{m_\mathrm{t} m_\mathrm{b}}\,,$$
$$K_{\mathrm{cs}} \simeq \mathrm{e}^{\mathrm{i}\sigma} + \sqrt{\frac{m_\mathrm{u} m_\mathrm{d}}{m_\mathrm{c} m_\mathrm{s}}} + \frac{m_\mathrm{c} m_\mathrm{s}}{m_\mathrm{t} m_\mathrm{b}}\mathrm{e}^{\mathrm{i}\tau},$$
$$K_{\mathrm{cb}} \simeq \frac{m_\mathrm{s}}{m_\mathrm{b}}\mathrm{e}^{\mathrm{i}\sigma} - \frac{m_\mathrm{c}}{m_\mathrm{t}}\mathrm{e}^{\mathrm{i}\tau},$$
$$K_{\mathrm{td}} \simeq \sqrt{\frac{m_\mathrm{d}}{m_\mathrm{s}}}\left(\frac{m_\mathrm{s}}{m_\mathrm{b}}\mathrm{e}^{\mathrm{i}\tau} - \frac{m_\mathrm{c}}{m_\mathrm{t}}\mathrm{e}^{\mathrm{i}\sigma}\right),$$
$$K_{\mathrm{ts}} \simeq \frac{m_\mathrm{c}}{m_\mathrm{t}}\mathrm{e}^{\mathrm{i}\sigma} - \frac{m_\mathrm{s}}{m_\mathrm{b}}\mathrm{e}^{\mathrm{i}\tau},$$
$$K_{\mathrm{tb}} \simeq \mathrm{e}^{\mathrm{i}\tau} + \frac{m_\mathrm{c} m_\mathrm{s}}{m_\mathrm{t} m_\mathrm{b}}\mathrm{e}^{\mathrm{i}\sigma}. \quad (28)$$

The best agreement of the $|K_{\mathrm{us}}|$ and $|K_{\mathrm{cd}}|$ elements with their experimental values is gotten when $\sigma = 81.09°$ and $\tau = 29.97°$. In this case we obtain $\psi_U = \psi_D + 81.09°$ and $\theta_U = \theta_D - 51.12°$. The numerical values of the CKM matrix elements are shown in Table 1. There is considerable disagreement between the magnitudes of the $|K_{\mathrm{ub}}|$, $|K_{\mathrm{cb}}|$, $|K_{\mathrm{td}}|$ and $|K_{\mathrm{ts}}|$ elements and their experimental values. On the other hand, the CP violating phase obtained for the Du–Xing ansatz is $\delta = 86.66°$. This result has an inconsistency with the experimental value [44] of about 30.58%.

Table 2 shows the error percentages of the CKM matrix elements and the CP violating phase δ for the different ansätze. The deviations of the CKM matrix elements obtained from the Du–Xing ansatz are higher than the corresponding ones of the Fritzsch ansatz. Therefore, the Fritzsch ansatz leads to a better prediction of the CKM matrix

Table 2. Error percentages of the CKM matrix elements and CP violating phase δ for the different ansätze considered. The experimental values used are those reported by [44]

	FA	DXA	DXFA	FDXA	$\widetilde{\mathrm{X}}$A	FNA	NTA
$\lvert K_{\mathrm{ud}}\rvert$	2.80	2.80	2.80	2.80	2.80	0.09	2.67
$\lvert K_{\mathrm{us}}\rvert$	0.00	0.00	0.00	0.00	0.00	1.60	0.00
$\lvert K_{\mathrm{ub}}\rvert$	50.32	341.21	46.03	10.30	118.31	28.72	108.64
$\lvert K_{\mathrm{cd}}\rvert$	0.00	0.05	0.00	0.00	0.03	1.65	0.04
$\lvert K_{\mathrm{cs}}\rvert$	2.84	2.89	2.84	2.84	2.88	0.09	2.75
$\lvert K_{\mathrm{cb}}\rvert$	72.79	141.69	70.44	38.58	18.50	0.00	32.01
$\lvert K_{\mathrm{td}}\rvert$	77.13	103.16	75.15	49.21	0.39	18.80	10.77
$\lvert K_{\mathrm{ts}}\rvert$	73.18	138.25	70.86	40.44	16.81	1.20	29.91
$\lvert K_{\mathrm{tb}}\rvert$	0.04	0.09	0.05	0.04	0.08	0.00	0.09
δ	30.54	30.58	30.55	30.58	26.79	168.72	25.43

elements than the Du–Xing ansatz. For the CP violating phase, the Fritzsch ansatz leads to approximately the same prediction as the one obtained from the Du–Xing ansatz.

On the other hand, by using the Du–Xing ansatz (26) and taking into account that $\xi^{U,D} = P\bar{\xi}^{U,D}P^{\dagger}$, we see that the Yukawa coupling matrices in the mass basis are

$$\xi^U \simeq \frac{\sqrt{2}}{v}\begin{pmatrix} \chi_{11}^{(U)} m_{\mathrm{u}} & \chi_{12}^{(U)}\sqrt{m_{\mathrm{u}}m_{\mathrm{c}}} & \chi_{13}^{(U)}\sqrt{m_{\mathrm{u}}m_{\mathrm{c}}} \\ \chi_{21}^{(U)}\sqrt{m_{\mathrm{u}}m_{\mathrm{c}}} & \chi_{22}^{(U)} m_{\mathrm{c}} & \chi_{23}^{(U)} m_{\mathrm{c}} \\ \chi_{31}^{(U)}\sqrt{m_{\mathrm{u}}m_{\mathrm{c}}} & \chi_{32}^{(U)} m_{\mathrm{c}} & \chi_{33}^{(U)} m_{\mathrm{t}} \end{pmatrix}, \tag{29}$$

$$\xi^D \simeq \frac{\sqrt{2}}{v}\begin{pmatrix} \chi_{11}^{(D)} m_{\mathrm{d}} & \chi_{12}^{(D)}\sqrt{m_{\mathrm{d}}m_{\mathrm{s}}} & \chi_{13}^{(D)}\sqrt{m_{\mathrm{d}}m_{\mathrm{s}}} \\ \chi_{21}^{(D)}\sqrt{m_{\mathrm{d}}m_{\mathrm{s}}} & \chi_{22}^{(D)} m_{\mathrm{s}} & \chi_{23}^{(D)} m_{\mathrm{s}} \\ \chi_{31}^{(D)}\sqrt{m_{\mathrm{d}}m_{\mathrm{s}}} & \chi_{32}^{(D)} m_{\mathrm{s}} & \chi_{33}^{(D)} m_{\mathrm{b}} \end{pmatrix}, \tag{30}$$

where

$$\begin{aligned}
\chi_{11}^{(U,D)} &= \left(c^{(U,D)} - 2d^{(U,D)}\right), \\
\chi_{12}^{(U,D)} &= \left(\chi_{21}^{(U,D)}\right)^* = \left(d^{(U,D)} - c^{(U,D)}\right)\mathrm{e}^{\mathrm{i}\psi_{U,D}}, \\
\chi_{22}^{(U,D)} &= c^{(U,D)}, &(31)\\
\chi_{13}^{(U,D)} &= \left(\chi_{31}^{(U,D)}\right)^* = \left(a^{(U,D)} - b^{(U,D)}\right)\mathrm{e}^{\mathrm{i}(\psi_{U,D}+\theta_{U,D})}, \\
\chi_{23}^{(U,D)} &= \left(\chi_{32}^{(U,D)}\right)^* = \left(b^{(U,D)} - a^{(U,D)}\right)\mathrm{e}^{\mathrm{i}\theta_{U,D}}, \\
\chi_{33}^{(U,D)} &= a^{(U,D)}. &(32)
\end{aligned}$$

Therefore, the Du–Xing ansatz leads to the same structure for the Yukawa couplings for the first and the second generation fermions. Also, the couplings to the third generation fermions in (29) and (30) are weaker than the ones predicted by the Fritzsch ansatz.

3.3 Combination of the Fritzsch and the Du–Xing ansatz

3.3.1 The Du–Xing ansatz for the up sector and the Fritzsch ansatz for the down sector (DFXA)

By using the Fritzsch and the Du–Xing ansätze for the down-type and up-type quarks, respectively, the following expressions corresponding to the CKM matrix elements are obtained:

$$\begin{aligned}
K_{\mathrm{ud}} &\simeq 1 - \mathrm{e}^{\mathrm{i}\sigma}\sqrt{\frac{m_{\mathrm{d}}m_{\mathrm{u}}}{m_{\mathrm{s}}m_{\mathrm{c}}}} - \mathrm{e}^{\mathrm{i}\tau}\frac{\sqrt{m_{\mathrm{u}}m_{\mathrm{c}}m_{\mathrm{d}}}}{m_{\mathrm{t}}\sqrt{m_{\mathrm{b}}}}, \\
K_{\mathrm{us}} &\simeq -\sqrt{\frac{m_{\mathrm{d}}}{m_{\mathrm{s}}}} - \mathrm{e}^{\mathrm{i}\sigma}\sqrt{\frac{m_{\mathrm{u}}}{m_{\mathrm{c}}}} - \mathrm{e}^{\mathrm{i}\tau}\frac{\sqrt{m_{\mathrm{u}}m_{\mathrm{c}}m_{\mathrm{s}}}}{m_{\mathrm{t}}\sqrt{m_{\mathrm{b}}}}, , \\
K_{\mathrm{ub}} &\simeq \mathrm{e}^{\mathrm{i}\tau}\frac{\sqrt{m_{\mathrm{u}}m_{\mathrm{c}}}}{m_{\mathrm{t}}} - \mathrm{e}^{\mathrm{i}\sigma}\sqrt{\frac{m_{\mathrm{u}}m_{\mathrm{s}}}{m_{\mathrm{c}}m_{\mathrm{b}}}}, \\
K_{\mathrm{cd}} &\simeq \mathrm{e}^{\mathrm{i}\sigma}\sqrt{\frac{m_{\mathrm{d}}}{m_{\mathrm{s}}}} + \sqrt{\frac{m_{\mathrm{u}}}{m_{\mathrm{c}}}} + \mathrm{e}^{\mathrm{i}\tau}\frac{m_{\mathrm{c}}}{m_{\mathrm{t}}}\sqrt{\frac{m_{\mathrm{d}}}{m_{\mathrm{b}}}}, \\
K_{\mathrm{cs}} &\simeq \mathrm{e}^{\mathrm{i}\sigma} + \mathrm{e}^{\mathrm{i}\tau}\frac{m_{\mathrm{c}}}{m_{\mathrm{t}}}\sqrt{\frac{m_{\mathrm{s}}}{m_{\mathrm{b}}}} - \sqrt{\frac{m_{\mathrm{d}}m_{\mathrm{u}}}{m_{\mathrm{s}}m_{\mathrm{c}}}}, \\
K_{\mathrm{cb}} &\simeq \mathrm{e}^{\mathrm{i}\sigma}\sqrt{\frac{m_{\mathrm{s}}}{m_{\mathrm{b}}}} - \mathrm{e}^{\mathrm{i}\tau}\frac{m_{\mathrm{c}}}{m_{\mathrm{t}}}, \\
K_{\mathrm{td}} &\simeq \mathrm{e}^{\mathrm{i}\sigma}\frac{m_{\mathrm{c}}}{m_{\mathrm{t}}}\sqrt{\frac{m_{\mathrm{d}}}{m_{\mathrm{s}}}} - \mathrm{e}^{\mathrm{i}\tau}\sqrt{\frac{m_{\mathrm{d}}}{m_{\mathrm{b}}}}, \\
K_{\mathrm{ts}} &\simeq \mathrm{e}^{\mathrm{i}\sigma}\frac{m_{\mathrm{c}}}{m_{\mathrm{t}}} - \mathrm{e}^{\mathrm{i}\tau}\sqrt{\frac{m_{\mathrm{s}}}{m_{\mathrm{b}}}}, \\
K_{\mathrm{tb}} &\simeq \mathrm{e}^{\mathrm{i}\tau} + \mathrm{e}^{\mathrm{i}\sigma}\frac{m_{\mathrm{c}}}{m_{\mathrm{t}}}\sqrt{\frac{m_{\mathrm{s}}}{m_{\mathrm{b}}}}. &(33)
\end{aligned}$$

With the aim to get better agreement of the $|K_{\mathrm{ud}}|$ and $|K_{\mathrm{cd}}|$ elements with their experimental values we obtain $\sigma = 81.09°$ and $\tau = 29.97°$, and then $\psi_U = \psi_D + 90°$ and $\theta_U = \theta_D - 51.12°$. The numerical values of the CKM matrix elements for this case are shown in Table 1. In this case, there is good agreement of the CKM matrix elements with their experimental values, with the exception of the $|K_{\mathrm{ub}}|$, $|K_{\mathrm{cb}}|$, $|K_{\mathrm{td}}|$ and $|K_{\mathrm{ts}}|$ elements, which present discrepancies of about 46.03%, 70.44%, 75.15% and 70.86%, respectively, to their corresponding experimental values. On the other hand, we obtain the result that the CP violating phase is $\delta = 86.62°$. This result presents a discrepancy of about 30.55%.

For the Yukawa coupling matrices in the mass basis for this case we have

$$\xi^U \simeq \frac{\sqrt{2}}{v}\begin{pmatrix} \chi_{11}^{(U)} m_{\mathrm{u}} & \chi_{12}^{(U)}\sqrt{m_{\mathrm{u}}m_{\mathrm{c}}} & \chi_{13}^{(U)}\sqrt{m_{\mathrm{u}}m_{\mathrm{c}}} \\ \chi_{21}^{(U)}\sqrt{m_{\mathrm{u}}m_{\mathrm{c}}} & \chi_{22}^{(U)} m_{\mathrm{c}} & \chi_{23}^{(U)} m_{\mathrm{c}} \\ \chi_{31}^{(U)}\sqrt{m_{\mathrm{u}}m_{\mathrm{c}}} & \chi_{32}^{(U)} m_{\mathrm{c}} & \chi_{33}^{(U)} m_{\mathrm{t}} \end{pmatrix}, \tag{34}$$

$$\xi^D \simeq \frac{\sqrt{2}}{v}\begin{pmatrix} \chi_{11}^{(D)} m_{\mathrm{d}} & \chi_{12}^{(D)}\sqrt{m_{\mathrm{d}}m_{\mathrm{s}}} & \chi_{13}^{(D)}\sqrt{m_{\mathrm{d}}m_{\mathrm{b}}} \\ \chi_{21}^{(D)}\sqrt{m_{\mathrm{d}}m_{\mathrm{s}}} & \chi_{22}^{(D)} m_{\mathrm{s}} & \chi_{23}^{(D)}\sqrt{m_{\mathrm{s}}m_{\mathrm{b}}} \\ \chi_{31}^{(D)}\sqrt{m_{\mathrm{d}}m_{\mathrm{b}}} & \chi_{32}^{(D)}\sqrt{m_{\mathrm{s}}m_{\mathrm{b}}} & \chi_{33}^{(D)} m_{\mathrm{b}} \end{pmatrix}, \tag{35}$$

where

$$\begin{aligned}
\chi_{11}^{(U)} &= \left(c^{(U)} - 2d^{(U)}\right), \\
\chi_{12}^{(U)} &= \left(\chi_{21}^{(U)}\right)^* = \left(d^{(U)} - c^{(U)}\right)\mathrm{e}^{\mathrm{i}\psi_U}, \\
\chi_{22}^{(U)} &= c^{(U)}, &(36)\\
\chi_{13}^{(U)} &= \left(\chi_{31}^{(U)}\right)^* = \left(a^{(U)} - b^{(U)}\right)\mathrm{e}^{\mathrm{i}(\psi_U+\theta_U)}, \\
\chi_{23}^{(U)} &= \left(\chi_{32}^{(U)}\right)^* = \left(b^{(U)} - a^{(U)}\right)\mathrm{e}^{\mathrm{i}\theta_U}, \\
\chi_{33}^{(U)} &= a^{(U)}, &(37)\\
\chi_{11}^{(D)} &= 2d^{(D)} - 2b^{(D)} + a^{(D)}, \\
\chi_{12}^{(D)} &= \left(\chi_{21}^{(D)}\right)^* = \left(a^{(D)} + d^{(D)} - 2b^{(D)}\right)\mathrm{e}^{\mathrm{i}\psi_D}, \\
\chi_{22}^{(D)} &= a^{(D)} - 2b^{(D)}, &(38)\\
\chi_{13}^{(D)} &= \left(\chi_{31}^{(D)}\right)^* = \left(b^{(D)} - a^{(D)}\right)\mathrm{e}^{\mathrm{i}(\psi_D+\theta_D)}, \\
\chi_{23}^{(D)} &= \left(\chi_{32}^{(D)}\right)^* = \left(b^{(D)} - a^{(D)}\right)\mathrm{e}^{\mathrm{i}\theta_D}, \\
\chi_{33}^{(D)} &= a^{(D)}. &(39)
\end{aligned}$$

3.3.2 The Fritzsch ansatz for the up quark sector and the Du–Xing ansatz for the down quark sector (FDXA)

In this section we use the Fritzsch and the Du–Xing ansätze for the up-type and down-type quarks, respectively. It is contrary to the DXFA case discussed before. The expressions for the CKM matrix elements are

$$\begin{aligned}
K_{\rm ud} &\simeq 1-{\rm e}^{{\rm i}\sigma}\sqrt{\frac{m_{\rm d}m_{\rm u}}{m_{\rm s}m_{\rm c}}}-{\rm e}^{{\rm i}\tau}\frac{\sqrt{m_{\rm u}m_{\rm d}m_{\rm s}}}{m_{\rm b}\sqrt{m_{\rm t}}}\,,\\
K_{\rm us} &\simeq \sqrt{\frac{m_{\rm d}}{m_{\rm s}}}+{\rm e}^{{\rm i}\sigma}\sqrt{\frac{m_{\rm u}}{m_{\rm c}}}+{\rm e}^{{\rm i}\tau}\frac{m_{\rm s}}{m_{\rm b}}\sqrt{\frac{m_{\rm u}}{m_{\rm t}}}\,,\\
K_{\rm ub} &\simeq {\rm e}^{{\rm i}\sigma}\frac{m_{\rm s}}{m_{\rm b}}\sqrt{\frac{m_{\rm u}}{m_{\rm c}}}-{\rm e}^{{\rm i}\tau}\sqrt{\frac{m_{\rm u}}{m_{\rm t}}}\,,\\
K_{\rm cd} &\simeq -{\rm e}^{{\rm i}\sigma}\sqrt{\frac{m_{\rm d}}{m_{\rm s}}}-\sqrt{\frac{m_{\rm u}}{m_{\rm c}}}-{\rm e}^{{\rm i}\tau}\frac{\sqrt{m_{\rm c}m_{\rm d}m_{\rm s}}}{m_{\rm b}\sqrt{m_{\rm t}}}\,,\\
K_{\rm cs} &\simeq {\rm e}^{{\rm i}\sigma}+{\rm e}^{{\rm i}\tau}\frac{m_{\rm s}}{m_{\rm b}}\sqrt{\frac{m_{\rm c}}{m_{\rm t}}}-\sqrt{\frac{m_{\rm d}m_{\rm u}}{m_{\rm s}m_{\rm c}}}\,,\\
K_{\rm cb} &\simeq {\rm e}^{{\rm i}\sigma}\frac{m_{\rm s}}{m_{\rm b}}-{\rm e}^{{\rm i}\tau}\sqrt{\frac{m_{\rm c}}{m_{\rm t}}}\,,\\
K_{\rm td} &\simeq {\rm e}^{{\rm i}\tau}\frac{\sqrt{m_{\rm d}m_{\rm s}}}{m_{\rm b}}-{\rm e}^{{\rm i}\sigma}\sqrt{\frac{m_{\rm d}m_{\rm c}}{m_{\rm s}m_{\rm t}}}\,,\\
K_{\rm ts} &\simeq {\rm e}^{{\rm i}\sigma}\sqrt{\frac{m_{\rm c}}{m_{\rm t}}}-{\rm e}^{{\rm i}\tau}\frac{m_{\rm s}}{m_{\rm b}}\,,\\
K_{\rm tb} &\simeq {\rm e}^{{\rm i}\tau}+{\rm e}^{{\rm i}\sigma}\frac{m_{\rm s}}{m_{\rm b}}\sqrt{\frac{m_{\rm c}}{m_{\rm t}}}\,.
\end{aligned}\tag{40}$$

In order to get the best agreement of the $|K_{\rm us}|$ and $|K_{\rm cd}|$ elements with their experimental values, we should have $\sigma=-98.93°$ and $\tau=52.19°$. $\psi_U=\psi_D-98.93°$ and $\theta_U=\theta_D+151.12°$ are found. Then, in accordance with the previous results, we have the numerical values presented in Table 1. In this case, there is good agreement of the CKM matrix elements with their experimental values, with the exception of the $|K_{\rm cb}|$, $|K_{\rm td}|$, $|K_{\rm ts}|$ elements, which present discrepancies of about 38.58%, 49.21%, and 40.44%, respectively, to their corresponding experimental values. On the other hand, we see that the CP violating phase for this case is $\delta=86.66°$. This result deviates by 30.58% from the experimental value.

In Table 2, the error percentages of the CKM matrix elements are lower than the ones corresponding to the $|K_{ij}|$ elements given by the DFXA case. This is an indication that, when the FDXA prescription is used, we obtain better agreement with the experimental values of the CKM matrix elements than in the combinations considered before.

On the other hand, the following Yukawa coupling matrices in the mass basis are obtained,

$$\xi^U\simeq\frac{\sqrt{2}}{v}\begin{pmatrix}\chi_{11}^{(U)}m_{\rm u} & \chi_{12}^{(U)}\sqrt{m_{\rm u}m_{\rm c}} & \chi_{13}^{(U)}\sqrt{m_{\rm u}m_{\rm t}}\\ \chi_{21}^{(U)}\sqrt{m_{\rm u}m_{\rm c}} & \chi_{22}^{(U)}m_{\rm c} & \chi_{23}^{(U)}\sqrt{m_{\rm c}m_{\rm t}}\\ \chi_{31}^{(U)}\sqrt{m_{\rm u}m_{\rm t}} & \chi_{32}^{(U)}\sqrt{m_{\rm c}m_{\rm t}} & \chi_{33}^{(U)}m_{\rm t}\end{pmatrix},\tag{41}$$

$$\xi^D\simeq\frac{\sqrt{2}}{v}\begin{pmatrix}\chi_{11}^{(D)}m_{\rm d} & \chi_{12}^{(D)}\sqrt{m_{\rm d}m_{\rm s}} & \chi_{13}^{(D)}\sqrt{m_{\rm d}m_{\rm s}}\\ \chi_{21}^{(D)}\sqrt{m_{\rm d}m_{\rm s}} & \chi_{22}^{(D)}m_{\rm s} & \chi_{23}^{(D)}m_{\rm s}\\ \chi_{31}^{(D)}\sqrt{m_{\rm d}m_{\rm s}} & \chi_{32}^{(D)}m_{\rm s} & \chi_{33}^{(D)}m_{\rm b}\end{pmatrix},\tag{42}$$

where

$$\begin{aligned}
\chi_{11}^{(U)}&=2d^{(U)}-2b^{(U)}+a^{(U)},\\
\chi_{12}^{(U)}&=\left(\chi_{21}^{(U)}\right)^*=(a^{(U)}+d^{(U)}-2b^{(U)}){\rm e}^{{\rm i}\psi_U},\\
\chi_{22}^{(U)}&=a^{(U)}-2b^{(U)},
\end{aligned}\tag{43}$$

$$\begin{aligned}
\chi_{13}^{(U)}&=\left(\chi_{31}^{(U)}\right)^*=(b^{(U)}-a^{(U)}){\rm e}^{{\rm i}(\psi_U+\theta_U)},\\
\chi_{23}^{(U)}&=\left(\chi_{32}^{(U)}\right)^*=(b^{(U)}-a^{(U)}){\rm e}^{{\rm i}\theta_U},\\
\chi_{33}^{(U)}&=a^{(U)},
\end{aligned}\tag{44}$$

$$\begin{aligned}
\chi_{11}^{(D)}&=(c^{(D)}-2d^{(D)}),\\
\chi_{12}^{(D)}&=\left(\chi_{21}^{(D)}\right)^*=(d^{(D)}-c^{(D)}){\rm e}^{{\rm i}\psi_D},\\
\chi_{22}^{(D)}&=c^{(D)},
\end{aligned}\tag{45}$$

$$\begin{aligned}
\chi_{13}^{(D)}&=\left(\chi_{31}^{(D)}\right)^*=(a^{(D)}-b^{(D)}){\rm e}^{{\rm i}(\psi_D+\theta_D)},\\
\chi_{23}^{(D)}&=\left(\chi_{32}^{(D)}\right)^*=(b^{(D)}-a^{(D)}){\rm e}^{{\rm i}\theta_D},\\
\chi_{33}^{(D)}&=a^{(D)}.
\end{aligned}\tag{46}$$

3.4 Combination of different assignments in the Du–Xing ansatz ($\widetilde{\rm X}$A)

It is known that taking $M^{\rm diag}={\rm diag}(-m_1,m_2,m_3)$, the following relations between the components of the $\widetilde{M}$ matrix given in (26) are satisfied [28, 29]:

$$\begin{aligned}
C+A&=-m_1+m_2+m_3\,,\\
CA-|B|^2-|D|^2&=-m_1m_2+m_2m_3+m_3m_1\,,\\
A|D|^2&=m_1m_2m_3\,.
\end{aligned}\tag{47}$$

The assumptions $A\gg|B|$ and $|C|\gg|D|$ define the assignment type A, and the assumption $C=m_2$ defines the assignment type B. For assignment type B, the parameters are

$$\begin{aligned}
A=m_3-m_1\,,\quad |D|&=\sqrt{\frac{m_1m_2m_3}{m_3-m_1}}\,,\\
|B|&=\sqrt{\frac{m_1m_3(m_3-m_2-m_1)}{m_3-m_1}}\,,
\end{aligned}\tag{48}$$

and the rotation matrix O is [18–21]

$$O\simeq\begin{pmatrix}1 & \sqrt{\frac{m_1}{m_2}} & \sqrt{\frac{m_2m_1^2}{m_3^3}}\\ -\sqrt{\frac{m_1}{m_2}} & 1 & \sqrt{\frac{m_1}{m_3}}\\ \sqrt{\frac{m_1^2}{m_2m_3}} & -\sqrt{\frac{m_1}{m_3}} & 1\end{pmatrix}\quad\text{for } m_3\gg m_2\gg m_1\,.\tag{49}$$

The assignments of type A and B are used for the up-type and down-type quarks, respectively. Then the following expressions for the CKM matrix elements are obtained:

$$
\begin{aligned}
K_{\mathrm{ud}} &\simeq 1+\mathrm{e}^{\mathrm{i}\sigma}\sqrt{\frac{m_{\mathrm{u}}m_{\mathrm{d}}}{m_{\mathrm{c}}m_{\mathrm{s}}}}+\mathrm{e}^{\mathrm{i}\tau}\frac{m_{\mathrm{d}}}{m_{\mathrm{t}}}\sqrt{\frac{m_{\mathrm{u}}m_{\mathrm{c}}}{m_{\mathrm{s}}m_{\mathrm{b}}}}\,,\\
K_{\mathrm{us}} &\simeq \sqrt{\frac{m_{\mathrm{d}}}{m_{\mathrm{s}}}}-\mathrm{e}^{\mathrm{i}\sigma}\sqrt{\frac{m_{\mathrm{u}}}{m_{\mathrm{c}}}}-\mathrm{e}^{\mathrm{i}\tau}\frac{\sqrt{m_{\mathrm{u}}m_{\mathrm{c}}m_{\mathrm{d}}}}{m_{\mathrm{t}}\sqrt{m_{\mathrm{b}}}}\,,\\
K_{\mathrm{ub}} &\simeq \sqrt{\frac{m_{\mathrm{s}}m_{\mathrm{d}}^2}{m_{\mathrm{b}}^3}}-\mathrm{e}^{\mathrm{i}\sigma}\sqrt{\frac{m_{\mathrm{u}}m_{\mathrm{d}}}{m_{\mathrm{c}}m_{\mathrm{b}}}}+\mathrm{e}^{\mathrm{i}\tau}\frac{\sqrt{m_{\mathrm{u}}m_{\mathrm{c}}}}{m_{\mathrm{t}}}\,,\\
K_{\mathrm{cd}} &\simeq \sqrt{\frac{m_{\mathrm{u}}}{m_{\mathrm{c}}}}-\frac{m_{\mathrm{c}}}{m_{\mathrm{t}}}\sqrt{\frac{m_{\mathrm{d}}^2}{m_{\mathrm{s}}m_{\mathrm{b}}}}\mathrm{e}^{\mathrm{i}\tau}-\mathrm{e}^{\mathrm{i}\sigma}\sqrt{\frac{m_{\mathrm{d}}}{m_{\mathrm{s}}}}\,,\\
K_{\mathrm{cs}} &\simeq \mathrm{e}^{\mathrm{i}\sigma}+\sqrt{\frac{m_{\mathrm{u}}m_{\mathrm{d}}}{m_{\mathrm{c}}m_{\mathrm{s}}}}+\mathrm{e}^{\mathrm{i}\tau}\frac{m_{\mathrm{c}}}{m_{\mathrm{t}}}\sqrt{\frac{m_{\mathrm{d}}}{m_{\mathrm{b}}}}\,,\\
K_{\mathrm{cb}} &\simeq \mathrm{e}^{\mathrm{i}\sigma}\sqrt{\frac{m_{\mathrm{d}}}{m_{\mathrm{b}}}}-\frac{m_{\mathrm{c}}}{m_{\mathrm{t}}}\mathrm{e}^{\mathrm{i}\tau}+\sqrt{\frac{m_{\mathrm{u}}m_{\mathrm{s}}m_{\mathrm{d}}^2}{m_{\mathrm{c}}m_{\mathrm{b}}^3}}\,,\\
K_{\mathrm{td}} &\simeq \mathrm{e}^{\mathrm{i}\tau}\sqrt{\frac{m_{\mathrm{d}}^2}{m_{\mathrm{s}}m_{\mathrm{b}}}}-\mathrm{e}^{\mathrm{i}\sigma}\frac{m_{\mathrm{c}}}{m_{\mathrm{t}}}\sqrt{\frac{m_{\mathrm{d}}}{m_{\mathrm{s}}}}\,,\\
K_{\mathrm{ts}} &\simeq \mathrm{e}^{\mathrm{i}\sigma}\frac{m_{\mathrm{c}}}{m_{\mathrm{t}}}-\mathrm{e}^{\mathrm{i}\tau}\sqrt{\frac{m_{\mathrm{d}}}{m_{\mathrm{b}}}}\,,\\
K_{\mathrm{tb}} &\simeq \mathrm{e}^{\mathrm{i}\tau}+\frac{m_{\mathrm{c}}}{m_{\mathrm{t}}}\sqrt{\frac{m_{\mathrm{d}}}{m_{\mathrm{b}}}}\mathrm{e}^{\mathrm{i}\sigma}.
\end{aligned}
\tag{50}
$$

In this case, $\sigma = 81.09°$, $\tau = -98.91°$ are obtained, getting good experimental agreement of the $|K_{\mathrm{us}}|$ and $|K_{\mathrm{cd}}|$ elements with their experimental values, and then $\psi_U = \psi_D + 81.09°$, $\theta_U = \theta_D - 180°$. The obtained numerical values for the CKM matrix elements are in Table 1. The CP violating phase is $\delta = 82.18°$, presenting a discrepancy of 26.79%. From Table 2, notice that the error percentages of the magnitudes of the CKM matrix elements are lower than 10.00% with the exception of the ones corresponding to the $|K_{\mathrm{ub}}|$, $|K_{\mathrm{cb}}|$ and $|K_{\mathrm{ts}}|$ elements, which are equal to 118.31%, 18.50% and 16.81%, respectively.

The corresponding Yukawa coupling matrices ξ^U and ξ^D in the mass basis for this case are given by

$$
\xi^U \simeq \frac{\sqrt{2}}{v}\begin{pmatrix} \chi_{11}^{(U)} m_{\mathrm{u}} & \chi_{12}^{(U)}\sqrt{m_{\mathrm{u}}m_{\mathrm{c}}} & \chi_{13}^{(U)}\sqrt{m_{\mathrm{u}}m_{\mathrm{c}}} \\ \chi_{21}^{(U)}\sqrt{m_{\mathrm{u}}m_{\mathrm{c}}} & \chi_{22}^{(U)} m_{\mathrm{c}} & \chi_{23}^{(U)} m_{\mathrm{c}} \\ \chi_{31}^{(U)}\sqrt{m_{\mathrm{u}}m_{\mathrm{c}}} & \chi_{32}^{(U)} m_{\mathrm{c}} & \chi_{33}^{(U)} m_{\mathrm{t}} \end{pmatrix}, \tag{51}
$$

$$
\xi^D \simeq \frac{\sqrt{2}}{v} \times \begin{pmatrix} \chi_{11}^{(D)} m_{\mathrm{d}} & \chi_{12}^{(D)}\sqrt{m_{\mathrm{d}}m_{\mathrm{s}}} & \chi_{13}^{(D)}\frac{m_{\mathrm{d}}}{m_{\mathrm{s}}}\sqrt{m_{\mathrm{s}}m_{\mathrm{b}}} \\ \chi_{21}^{(D)}\sqrt{m_{\mathrm{d}}m_{\mathrm{s}}} & \chi_{22}^{(D)} m_{\mathrm{s}} & \chi_{23}^{(D)}\sqrt{m_{\mathrm{d}}m_{\mathrm{b}}} \\ \chi_{31}^{(D)}\frac{m_{\mathrm{d}}}{m_{\mathrm{s}}}\sqrt{m_{\mathrm{s}}m_{\mathrm{b}}} & \chi_{32}^{(D)}\sqrt{m_{\mathrm{d}}m_{\mathrm{b}}} & \chi_{33}^{(D)} m_{\mathrm{b}} \end{pmatrix}, \tag{52}
$$

where

$$
\begin{aligned}
\chi_{11}^{(U)} &= (c^{(U)} - 2d^{(U)})\,,\\
\chi_{12}^{(U)} &= \left(\chi_{21}^{(U)}\right)^* = (d^{(U)} - c^{(U)})\mathrm{e}^{\mathrm{i}\psi_U}\,,\\
\chi_{22}^{(U)} &= c^{(U)}, \qquad (53)\\
\chi_{13}^{(U)} &= \left(\chi_{31}^{(U)}\right)^* = (a^{(U)} - b^{(U)})\mathrm{e}^{\mathrm{i}(\psi_U+\theta_U)}\,,\\
\chi_{23}^{(U)} &= \left(\chi_{32}^{(U)}\right)^* = (b^{(U)} - a^{(U)})\mathrm{e}^{\mathrm{i}\theta_U}\,,\\
\chi_{33}^{(U)} &= a^{(U)}, \qquad (54)\\
\chi_{11}^{(D)} &= (c^{(D)} - 2d^{(D)})\,,\\
\chi_{12}^{(D)} &= \left(\chi_{21}^{(D)}\right)^* = (d^{(D)} - c^{(D)})\mathrm{e}^{\mathrm{i}\psi_D}\,,\\
\chi_{22}^{(D)} &= c^{(D)}, \qquad (55)\\
\chi_{13}^{(D)} &= \left(\chi_{31}^{(D)}\right)^* = a^{(D)}\mathrm{e}^{\mathrm{i}(\psi_D+\theta_D)}\,,\\
\chi_{23}^{(D)} &= \left(\chi_{32}^{(D)}\right)^* = (b^{(D)} - a^{(D)})\mathrm{e}^{\mathrm{i}\theta_D}\,,\\
\chi_{33}^{(D)} &= a^{(D)}. \qquad (56)
\end{aligned}
$$

From (51) and (52), we observe that the Yukawa couplings to the first and the second generation fermions have the same form as predicted by the Fritzsch and the Du–Xing ansätze. The couplings to the third generation fermions are stronger than the ones predicted by the Du–Xing ansatz and weaker than the ones obtained by the Cheng and Sher ansatz. This fact has consequences in the phenomenology, as we will show in Sect. 4.

3.5 Fukuyama–Nishiura ansatz (FNA)

Recently, the following mass matrix ansatz has been proposed for quarks and leptons [18–21]:

$$
\widetilde{M} = \begin{pmatrix} 0 & A & A \\ A & B & C \\ A & C & B \end{pmatrix}. \tag{57}
$$

This form has originally been used for leptons (neutrinos) in order to reproduce nearly bi-maximal lepton mixing. Moreover, the mass matrix $\widetilde{M}$ is diagonalized by a rotation matrix according to

$$
O^{\mathrm{T}}\widetilde{M}O = \mathrm{diag}(-m_1, m_2, m_3)\,, \tag{58}
$$

where m_1, m_2 and m_3 correspond to the fermion masses with $m_1 \ll m_2 \ll m_3$. By taking into account that $\mathrm{tr}(\widetilde{M})$ and $\det(\widetilde{M})$ are invariants, and using the eigenvalue equation for $\widetilde{M}$, we get

$$
\begin{aligned}
A &= \pm\sqrt{\frac{m_1 m_2}{2}}\,, \quad B = \frac{1}{2}(m_2 + m_3 - m_1)\,,\\
C &= -\frac{1}{2}(m_3 - m_2 + m_1)\,.
\end{aligned}
\tag{59}
$$

The case where $B - C$ takes its maximum value corresponds to assignment type C. Assignment type D is obtained by exchanging m_2 and m_3 in the already mentioned type C; see (59). For assignment type D,

$$A = \pm\sqrt{\frac{m_3 m_1}{2}}\,, \quad B = \frac{1}{2}(m_3 + m_2 - m_1)\,,$$
$$C = \frac{1}{2}(m_3 - m_2 - m_1)\,. \tag{60}$$

The assignments of types C and D are used for the down-type and up-type quarks, respectively. The rotation matrices that diagonalize $\widetilde{M}$ for assignments of types C and D are

$$O = \begin{pmatrix} c & s & 0 \\ -\frac{s}{\sqrt{2}} & \frac{c}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{s}{\sqrt{2}} & \frac{c}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad O' = \begin{pmatrix} c' & 0 & s' \\ -\frac{s'}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{c'}{\sqrt{2}} \\ -\frac{s'}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{c'}{\sqrt{2}} \end{pmatrix} \tag{61}$$

respectively, where

$$c = \cos\varphi = \sqrt{\frac{m_2}{m_2 + m_1}}\,, \qquad s = \sin\varphi = \sqrt{\frac{m_1}{m_1 + m_2}}\,,$$
$$c' = \cos\varphi' = \pm\sqrt{\frac{m_3}{m_1 + m_3}}\,, \quad s' = \sin\varphi' = \pm\sqrt{\frac{m_1}{m_1 + m_3}}\,.$$

In order to get good agreement of $\frac{1}{9}\sum_{i=1}^{9}|K_{ij}|$ with their experimental values, we should have $\sigma = -180°$ and $\tau = 4.84°$. From this result, the numerical values corresponding to the updated magnitudes of the CKM matrix elements for the Fukuyama–Nishiura ansatz are obtained [18–21] (see Table 1).

From Table 1, we see that there is very good agreement of the CKM matrix elements with their experimental values, since the corresponding error percentages are lower than 2%. Exceptions are the $|K_{\mathrm{ub}}|$ and $|K_{\mathrm{td}}|$ elements, which present discrepancies of about 28.72% and 18.80%, respectively, to their corresponding experimental magnitudes. It is also important to point out that the Fukuyama–Nishiura ansatz leads to a much better prediction of the CKM matrix elements than the ones obtained from the Fritzsch and the Du–Xing ansätze. The CP violating phase $\delta = -87.55°$ in this case is inconsistent with its experimental value, because it presents a discrepancy of about 168.72%. Furthermore, $\theta_U = \theta_D + 184.84$ and $\psi_U = \psi_D - 4.84°$ are obtained.

Finally, the Yukawa coupling matrices in the mass basis are given by

$$\xi^U \simeq \frac{\sqrt{2}}{2v} \times \begin{pmatrix} \chi_{11}^{(U)} m_{\mathrm{u}} & 0 & \chi_{13}^{(U)}\sqrt{m_{\mathrm{u}} m_{\mathrm{t}}} \\ 0 & \chi_{22}^{\prime(U)} m_{\mathrm{t}} + \chi_{22}^{(U)} m_{\mathrm{c}} & 0 \\ \chi_{31}^{(U)}\sqrt{m_{\mathrm{u}} m_{\mathrm{t}}} & 0 & \chi_{33}^{(U)} m_{\mathrm{t}} \end{pmatrix}, \tag{62}$$

$$\xi^D \simeq \frac{\sqrt{2}}{2v}\begin{pmatrix} \chi_{11}^{(D)} m_{\mathrm{d}} & \chi_{12}^{(D)}\sqrt{\frac{m_{\mathrm{d}}}{m_{\mathrm{s}}}} m_{\mathrm{b}} & 0 \\ \chi_{21}^{(D)}\sqrt{\frac{m_{\mathrm{d}}}{m_{\mathrm{s}}}} m_{\mathrm{b}} & \chi_{22}^{\prime(D)} m_{\mathrm{b}} + \chi_{22}^{(D)} m_{\mathrm{s}} & 0 \\ 0 & 0 & \chi_{33}^{(D)} m_{\mathrm{b}} \end{pmatrix}, \tag{63}$$

where

$$\chi_{11}^{(U)} = b^{(U)} + c^{(U)} - 4a^{(U)},$$
$$\chi_{22}^{(U)} = \chi_{33}^{(U)} = b^{(U)} + c^{(U)}, \tag{64}$$
$$\chi_{13}^{(U)} = \left(\chi_{31}^{(U)}\right)^* = (2a^{(U)} + b^{(U)} - c^{(U)})\mathrm{e}^{\mathrm{i}(\psi_U + \theta_U)},$$
$$\chi_{22}^{\prime(U)} = b^{(U)} - c^{(U)}, \tag{65}$$
$$\chi_{11}^{(D)} = 2(b^{(D)} - 2a^{(D)}),$$
$$\chi_{22}^{(D)} = \chi_{33}^{(D)} = b^{(D)} + c^{(D)}, \tag{66}$$
$$\chi_{12}^{(D)} = \left(\chi_{21}^{(U)}\right)^* = (b^{(D)} - c^{(D)})\mathrm{e}^{\mathrm{i}\psi_D},$$
$$\chi_{22}^{\prime(D)} = b^{(D)} - c^{(D)}. \tag{67}$$

From the above expressions, the couplings between the neutral Higgs field h^0 and the quark pairs u–c, c–t, d–b and s–b are vanishing. This means that there are no flavor changing neutral currents involving terms like $h^0\bar{\mathrm{u}}\mathrm{c}$, $h^0\bar{\mathrm{c}}\mathrm{t}$, $h^0\bar{\mathrm{d}}\mathrm{b}$, $h^0\bar{\mathrm{s}}\mathrm{b}$.

3.6 Non-mixing top quark ansatz (NTA)

We propose the following mass matrix ansatz for the up-type quarks:

$$\widetilde{M} = \begin{pmatrix} B & C & 0 \\ C & D & 0 \\ 0 & 0 & A \end{pmatrix}, \tag{68}$$

which is diagonalized by a rotation matrix O given by

$$O = \begin{pmatrix} c_\phi & -s_\phi & 0 \\ s_\phi & c_\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{69}$$

in accordance with $O^{\mathrm{T}}\widetilde{M}O = \mathrm{diag}(m_1, -m_2, m_3)$. In this case the condition $\tan\phi = \sqrt{\frac{m_2}{m_3}}$ has been posed, and using the expressions

$$\det\widetilde{M} = A(BD - C^2)\,, \quad \mathrm{tr}\,\widetilde{M} = B + D = -m_2 + m_1\,,$$

we obtain

$$A = m_3\,, \qquad B = \frac{m_1 m_3 - m_2^2}{m_2 + m_3}\,,$$
$$C = \frac{m_1 + m_2}{m_2 + m_3}\sqrt{m_2 m_3}\,, \quad D = m_2\frac{m_1 - m_3}{m_2 + m_3}\,, \tag{70}$$

and, therefore, the rotation matrix is

$$O = \begin{pmatrix} \sqrt{\frac{m_3}{m_2+m_3}} & -\sqrt{\frac{m_2}{m_2+m_3}} & 0 \\ \sqrt{\frac{m_2}{m_2+m_3}} & \sqrt{\frac{m_3}{m_2+m_3}} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{71}$$

By using the mass matrix ansatz given by (68) for the up-type quarks and the assignment of type B of the Du–Xing ansatz for the down-type quarks (26), the following CKM matrix elements are obtained:

$$\begin{aligned}
K_{\mathrm{ud}} &\simeq c_U - s_U\sqrt{\frac{m_{\mathrm{d}}}{m_{\mathrm{s}}}}\mathrm{e}^{\mathrm{i}\sigma},\\
K_{\mathrm{us}} &\simeq c_U\sqrt{\frac{m_{\mathrm{d}}}{m_{\mathrm{s}}}} + s_U\mathrm{e}^{\mathrm{i}\sigma},\\
K_{\mathrm{ub}} &\simeq c_U\sqrt{\frac{m_{\mathrm{d}}^2 m_{\mathrm{s}}}{m_{\mathrm{b}}^3}} + s_U\sqrt{\frac{m_{\mathrm{d}}}{m_{\mathrm{b}}}}\mathrm{e}^{\mathrm{i}\sigma},\\
K_{\mathrm{cd}} &\simeq -s_U - c_U\sqrt{\frac{m_{\mathrm{d}}}{m_{\mathrm{s}}}}\mathrm{e}^{\mathrm{i}\sigma},\\
K_{\mathrm{cs}} &\simeq c_U\mathrm{e}^{\mathrm{i}\sigma} - s_U\sqrt{\frac{m_{\mathrm{d}}}{m_{\mathrm{s}}}},\\
K_{\mathrm{cb}} &\simeq c_U\sqrt{\frac{m_{\mathrm{d}}}{m_{\mathrm{b}}}}\mathrm{e}^{\mathrm{i}\sigma} - s_U\sqrt{\frac{m_{\mathrm{d}}^2 m_{\mathrm{s}}}{m_{\mathrm{b}}^3}},\\
K_{\mathrm{td}} &\simeq \sqrt{\frac{m_{\mathrm{d}}^2}{m_{\mathrm{s}} m_{\mathrm{b}}}}\mathrm{e}^{\mathrm{i}\tau},\\
K_{\mathrm{ts}} &\simeq -\mathrm{e}^{\mathrm{i}\tau}\sqrt{\frac{m_{\mathrm{d}}}{m_{\mathrm{b}}}},\\
K_{\mathrm{tb}} &\simeq \mathrm{e}^{\mathrm{i}\tau},
\end{aligned} \tag{72}$$

where

$$c_U = \sqrt{\frac{m_{\mathrm{t}}}{m_{\mathrm{c}} + m_{\mathrm{t}}}}, \quad s_U = \sqrt{\frac{m_{\mathrm{c}}}{m_{\mathrm{c}} + m_{\mathrm{t}}}}.$$

In order to get the best agreement of the $|K_{\mathrm{us}}|$ and $|K_{\mathrm{cd}}|$ elements with their experimental values, we should have $\sigma = -99.24°$. On the other hand, by adjusting the τ parameter in order to get the best agreement of $\mathrm{Im}\, m(K_{\mathrm{ts}})$ with its experimental value, we should have $\tau = 90.00°$. Therefore we obtain the magnitudes of the CKM matrix elements shown in Table 1. Moreover, for this case we have $\psi_U = \psi_D - 99.24°$, $\theta_U = \theta_D + 189.24°$. For the CP violating phase we obtain $\delta = 80.68°$. This result roughly agrees with the experimental value, presenting a discrepancy of about 25.43%.

Table 2 shows the error percentages of the magnitudes of the Cabibbo–Kobayashi–Maskawa matrix elements and the CP violating phase δ with respect to their experimental values. In this table we observe that the error percentages of the magnitudes of the CKM matrix elements are lower than 11.00% with the exception of the ones corresponding to the $|K_{\mathrm{ub}}|$, $|K_{\mathrm{cb}}|$ and $|K_{\mathrm{ts}}|$ elements, which are equal to 108.64%, 32.01% and 29.91%, respectively. For this reason, the Cabibbo–Kobayashi–Maskawa matrix obtained by using the above prescription exhibits good agreement with the experimental results.

According to the mass matrix structure given by (68), we propose the following ansatz for the Yukawa coupling matrix in the flavor basis:

$$\xi^{U,0} = \frac{\sqrt{2}}{v}\begin{pmatrix} b^{(U)}B_U & c^{(U)}C_U\mathrm{e}^{\mathrm{i}\psi_U} & 0 \\ c^{(U)}C_U\mathrm{e}^{-\mathrm{i}\psi_U} & d^{(U)}D_U & 0 \\ 0 & 0 & a^{(U)}A_U \end{pmatrix}. \tag{73}$$

Hence, when the new ansatz and type B assignment of the Du–Xing ansatz are used for the quarks U and D, respectively, the Yukawa couplings matrices in the mass basis are given by

$$\xi^U = \frac{\sqrt{2}}{v}\begin{pmatrix} \chi_{11}^{(U)} m_{\mathrm{u}} & \chi_{12}^{(U)}\frac{m_{\mathrm{c}}}{m_{\mathrm{t}}}\sqrt{m_{\mathrm{c}} m_{\mathrm{t}}} & 0 \\ \chi_{21}^{(U)}\frac{m_{\mathrm{c}}}{m_{\mathrm{t}}}\sqrt{m_{\mathrm{c}} m_{\mathrm{t}}} & \chi_{22}^{(U)} m_{\mathrm{c}} & 0 \\ 0 & 0 & \chi_{33}^{(U)} m_{\mathrm{t}} \end{pmatrix}, \tag{74}$$

$$\xi^D \simeq \frac{\sqrt{2}}{v} \times \begin{pmatrix} \chi_{11}^{(D)} m_{\mathrm{d}} & \chi_{12}^{(D)}\sqrt{m_{\mathrm{d}} m_{\mathrm{s}}} & \chi_{13}^{(D)}\frac{m_{\mathrm{d}}}{m_{\mathrm{s}}}\sqrt{m_{\mathrm{s}} m_{\mathrm{b}}} \\ \chi_{21}^{(D)}\sqrt{m_{\mathrm{d}} m_{\mathrm{s}}} & \chi_{22}^{(D)} m_{\mathrm{s}} & \chi_{23}^{(D)}\sqrt{m_{\mathrm{d}} m_{\mathrm{b}}} \\ \chi_{31}^{(D)}\frac{m_{\mathrm{d}}}{m_{\mathrm{s}}}\sqrt{m_{\mathrm{s}} m_{\mathrm{b}}} & \chi_{32}^{(D)}\sqrt{m_{\mathrm{d}} m_{\mathrm{b}}} & \chi_{33}^{(D)} m_{\mathrm{b}} \end{pmatrix}, \tag{75}$$

where

$$\begin{aligned}
&\chi_{11}^{(U)} = b^{(U)}, \quad \chi_{22}^{(U)} = -d^{(U)}, \quad \chi_{33}^{(U)} = a^{(U)},\\
&\chi_{12}^{(U)} = \left(\chi_{21}^{(U)}\right)^* = (c^{(U)} - d^{(U)})\mathrm{e}^{\mathrm{i}\psi_U},
\end{aligned} \tag{76}$$

$$\begin{aligned}
&\chi_{11}^{(D)} = (c^{(D)} - 2d^{(D)}),\\
&\chi_{12}^{(D)} = \left(\chi_{21}^{(D)}\right)^* = (d^{(D)} - c^{(D)})\mathrm{e}^{\mathrm{i}\psi_D},\\
&\chi_{22}^{(D)} = c^{(D)},
\end{aligned} \tag{77}$$

$$\begin{aligned}
&\chi_{13}^{(D)} = \left(\chi_{31}^{(D)}\right)^* = a^{(D)}\mathrm{e}^{\mathrm{i}(\psi_D + \theta_D)},\\
&\chi_{23}^{(D)} = \left(\chi_{32}^{(D)}\right)^* = (b^{(D)} - a^{(D)})\mathrm{e}^{\mathrm{i}\theta_D},\\
&\chi_{33}^{(D)} = a^{(D)}.
\end{aligned} \tag{78}$$

From the previous expressions, the couplings between the neutral Higgs field h^0 and the quarks pairs u–t and c–t are vanishing, which implies that there are no flavor changing neutral currents involving terms of the form $h^0\bar{\mathrm{u}}\mathrm{t}$, $h^0\bar{\mathrm{c}}\mathrm{t}$.

4 Two body decays of the top quark, h^0 and $\mathrm{H}^\pm$

Consider the top quark decay to an up-type quark plus the lightest neutral Higgs boson h^0. The interest in FCNC is expected to increase since this issue will be examined at both the LHC and ILC, where one hopes to reach sensibilities of the order of $B(\mathrm{t} \to qh^0) \gtrsim 10^{-5}$. In the framework of the SM, the branching fractions are strongly suppressed, $B(\mathrm{t} \to \mathrm{c}h^0) \sim 10^{-15}$ and $B(\mathrm{t} \to \mathrm{u}h^0) \sim 10^{-17}$. In the framework of 2HDM type III, enhancement is expected due to the presence of FCNC at tree level. The decay width takes the form

$$\Gamma(\mathrm{t} \to qh^0) = \frac{G_{\mathrm{F}} m_{\mathrm{t}}}{4\sqrt{2}\pi}\left(1 - \frac{m_h^2}{m_{\mathrm{t}}^2}\right)^2 \left|\lambda_{\mathrm{t}q}^U\right|^2 \cos^2\alpha\,, \tag{79}$$

where $\lambda_{\mathrm{t}q}^U$ is the coupling $\mathrm{t}qh^0$. Therefore, there are two possible decays: $\mathrm{t} \to \mathrm{u}h^0$ and $\mathrm{t} \to \mathrm{c}h^0$.

In Fig. 1, the branching fraction $B(\mathrm{t} \to ch^0)$ as a function of the lightest neutral Higgs boson mass for the Fritzsch ansatz is plotted and results of the order of 10^{-3} are gotten. In Fig. 1, a decrease of the branching ratio with the increase of the Higgs mass is observed, but notice that the branching ratio rises with increasing magnitudes of the coefficients $a^{(U)} - b^{(U)}$ of the coupling $\mathrm{t}ch^0$. Also, the branching ratio decreases with the reduction of the mixing angle α between the neutral Higgs fields h^0 and H^0. These coefficients and the mixing angle α were taken to be equal to 0.75, 1.00 and $\frac{\pi}{4}$, $\frac{\pi}{15}$ and $\frac{4\pi}{9}$, respectively. The branching fraction $B(\mathrm{t} \to \mathrm{u}h^0)$ is two orders of magnitude lower than $B(\mathrm{t} \to \mathrm{c}h^0)$, and its behavior is quite similar.

In order to consider the different ansätze, the following ratio is useful:

$$R^{(A/\mathrm{F})} = \frac{B(\mathrm{t} \to qh^0)_A}{B(\mathrm{t} \to qh^0)_\mathrm{F}} = \left| \frac{\lambda^U_{\mathrm{t}q,A}}{\lambda^U_{\mathrm{t}q,\mathrm{F}}} \right|^2 , \qquad (80)$$

which is with respect to the Fritzsch ansatz (F), with $q = \mathrm{u}$, c and $A = $ DXA, FDXA, DXFA, $\widetilde{\mathrm{X}}$A, FNA, NTA. It does not depend on m_t, m_h or $\cos\alpha$, but it depends on the explicit form of the masses in the different kind of ansätze discussed. This ratio is typically of the order of 10^{-3}, but in the case of the FNA and NTA such a kind of top quark decay width is zero, because the top quark is completely decoupled from the two lighter generations.

Now, let us turn attention to decays of the lightest neutral Higgs boson h^0. The h^0 decays to the quark pairs $\mathrm{d\bar{b}}$, $\mathrm{s\bar{b}}$, $\mathrm{d\bar{s}}$ and $\mathrm{b\bar{b}}$ are considered. In the framework of the SM, the channel $h^0_\mathrm{SM} \to \mathrm{b\bar{b}}$ is the channel most studied, because it plays a central role in the possible detection in the range 120–140 GeV, and its signature will be clear enough. On the other hand, decays like $h^0 \to \mathrm{s\bar{b}}$ are interesting to study, because FCNC in the Higgs sector would be clear evidence of physics beyond the SM. In order to look for differences, the ratio between the decay widths $\Gamma(h^0 \to \mathrm{b\bar{b}})_A$ and $\Gamma(h^0 \to \mathrm{b\bar{b}})_\mathrm{SM}$ is defined according to

$$r_\mathrm{bb} = \frac{\Gamma(h^0 \to \mathrm{b\bar{b}})_A}{\Gamma(h^0 \to \mathrm{b\bar{b}})_\mathrm{SM}} . \qquad (81)$$

Since for all ansätze considered in this work, the Yukawa couplings of the neutral Higgs boson h^0 and the quark pair $\mathrm{b\bar{b}}$ have the same intensities; the ratio r_bb has the same value for all ansätze. However, the ratio r_bb gets the maximum value when $\cot\alpha = -a^D$; then for $\alpha = -\pi/4$ and $a^D = 1$, the ratio is 2, and this means that the decay width for these ansätze can be twice the decay width predicted by the SM.

Another possible definition is the ratio

$$\gamma^{(A)}_\mathrm{sb} = \frac{\Gamma(h^0 \to \mathrm{s\bar{b}})_A}{\Gamma(h^0 \to \mathrm{b\bar{b}})_A} . \qquad (82)$$

From Table 3, the relative decay widths for the process $h^0 \to \mathrm{s\bar{b}}$ predicted by the Fritzsch ansatz are at least one order of magnitude bigger than the ones corresponding to the assignments of type A and type B of the Du–Xing ansatz. This is due to the fact that the intensity of the

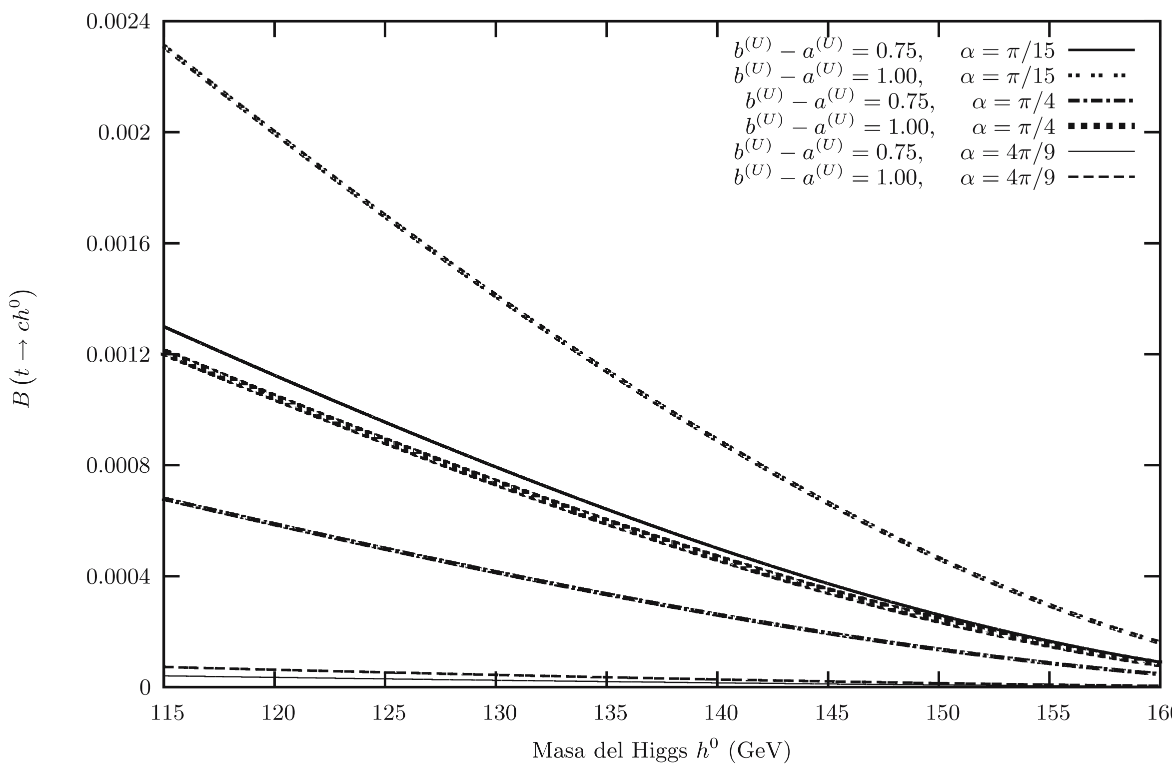

Fig. 1. Branching ratio $B(\mathrm{t} \to \mathrm{ch}^0)$ for the process $\mathrm{t} \to \mathrm{ch}^0$ in the Fritzsch ansatz and using different values of the Yukawa coupling parameters and α

Table 3. Relative decay width $\gamma_{\rm sb}^{(A)} = \frac{\Gamma(h^0 \to {\rm s\bar{b}})_A}{\Gamma(h^0 \to {\rm b\bar{b}})_A}$ for the process $h^0 \to {\rm s\bar{b}}$ in the different ansätze with $M_{h^0} = 120$ GeV and the Yukawa coupling coefficients of the order of one

	$\alpha = \frac{\pi}{3}$	$\alpha = \frac{\pi}{18}$	$\alpha = \frac{4\pi}{9}$
$\gamma_{\rm sb}^{(\rm FA)}$	7.29×10^{-2}	2.88×10^{-2}	8.95×10^{-4}
$\gamma_{\rm sb}^{(\rm DXA)}$	7.10×10^{-4}	5.61×10^{-4}	4.71×10^{-5}
$\gamma_{\rm sb}^{(\rm FDXA)}$	7.10×10^{-4}	5.61×10^{-4}	4.71×10^{-5}
$\gamma_{\rm sb}^{(\rm DXFA)}$	7.29×10^{-2}	2.88×10^{-2}	8.95×10^{-4}
$\gamma_{\rm sb}^{(\widetilde{\rm X}\rm A)}$	1.92×10^{-3}	1.51×10^{-3}	4.71×10^{-5}

Yukawa couplings to the third generation fermions for the Fritzsch ansatz is stronger than the one corresponding to the assignments of type A and type B of the Du–Xing ansatz, which implies that the Fritzsch ansatz leads to a higher probability of finding the decay processes $h^0 \to {\rm s\bar{b}}$ than the ones obtained by these assignments of the Du–Xing ansatz.

Now, for the decays of the charged Higgs boson, two cases are considered: a charged Higgs boson lighter than the top quark, $m_H = 150$ GeV, and a heavier charged Higgs boson, $m_H = 250$ GeV. In the first case, the top quark could decay into a charged Higgs boson, and this will be an alternative to the usual channel ${\rm t} \to {\rm W^+ b}$. Taking a light charged Higgs boson, the interesting channels to detect a Higgs boson would be $H^+ \to {\rm c\bar{s}}$, $H^+ \to {\rm c\bar{b}}$ and ${\rm t} \to H^+{\rm b}$. The decay widths were evaluated using phases different from zero, $\psi_D = \theta_D = \frac{\pi}{18}$, but we also made an evaluation with phases equal to zero, and the results do not change in order of magnitude. In Table 4 the results are shown. Notice that the channel $H^+ \to {\rm c\bar{b}}$ could compete with the channel searched, $H^+ \to {\rm c\bar{s}}$; however, in the FNA and NTA ansätze the ${\rm c\bar{s}}$ channel is bigger than the ${\rm c\bar{b}}$ channel. As regards the option ${\rm t} \to H^+{\rm b}$, the branching fraction is of the order of 10^{-1} in the different ansätze considered.

In the case of a charged Higgs boson heavier than the top quark, $\Gamma(H^+ \to {\rm t\bar{b}})$, $\Gamma(H^+ \to {\rm t\bar{s}})$, $\Gamma(H^+ \to {\rm t\bar{d}})$, $\Gamma(H^+ \to {\rm c\bar{s}})$ and $\Gamma(H^+ \to {\rm c\bar{b}})$ have been evaluated, and they are shown in Table 5. It is worth to notice that the most important channel would be $H^+ \to {\rm t\bar{b}}$, except for the FNA, where it is $H^+ \to {\rm c\bar{s}}$.

5 Conclusions

The quark mass matrices ansätze proposed by Fritzsch, Du–Xing and Fukuyama–Nishiura [1–4, 13, 28, 29] have been reviewed in the framework of the general two Higgs doublet model and the corresponding Yukawa matrices in the flavor basis. For these ansätze, the numerical values of the CKM matrix elements and their experimental values have been compared, obtaining the result that the Fukuyama–Nishiura ansatz leads to the best agreement with the experimental results. The CKM matrix elements have also been obtained by combining the Fritzsch and the Du–Xing ansätze for the up-type and down-type quarks in different ways. For the CKM matrix obtained by using the Fritzsch and the Du–Xing ansätze for the U-type and D-type quarks, respectively, better agreement with the experimental CKM matrix elements is gotten than the ones resulting when other Fritzsch and Du–Xing ansätze combinations are used. Moreover, the CKM matrix by using two different assignments of the Du–Xing ansatz for the U-type and D-type quarks has been computed, obtaining very good consistency between the magnitudes of six of their elements and their experimental values. In the Fukuyama–Nishiura ansatz, the Yukawa coupling matrices in the mass basis for both types of quarks have been computed by using two different assignments for the U-

Table 4. The decay widths $\Gamma({\rm H^+} \to {\rm c\bar{s}})$, $\Gamma({\rm H^+} \to {\rm c\bar{b}})$ and the fraction $B({\rm t} \to {\rm H^+ b})$ using the mass matrices ansätze discussed and setting up $M_{\rm H^+} = 150$ GeV, $\psi_D = \theta_D = \frac{\pi}{18}$

Γ (GeV)	FA	DXA	FDXA	DXFA	$\widetilde{\rm X}$A	FNA	NTA
$\Gamma({\rm H^+} \to {\rm c\bar{s}})$	4.83×10^{-4}	3.77×10^{-5}	6.11×10^{-5}	3.34×10^{-5}	3.49×10^{-5}	2.81	3.72×10^{-5}
$\Gamma({\rm H^+} \to {\rm c\bar{b}})$	1.04×10^{-2}	3.87×10^{-5}	1.06×10^{-2}	1.12×10^{-4}	4.41×10^{-5}	5.28×10^{-3}	3.15×10^{-6}
$B({\rm t} \to {\rm H^+ b})$	1.22×10^{-1}	1.24×10^{-1}	1.24×10^{-1}	1.24×10^{-1}	1.24×10^{-1}	1.24×10^{-1}	1.24×10^{-1}

Table 5. The decay widths $\Gamma({\rm H^+} \to {\rm c\bar{s}})$, $\Gamma({\rm H^+} \to {\rm c\bar{b}})$, $\Gamma({\rm H^+} \to {\rm t\bar{b}})$, $\Gamma({\rm H^+} \to {\rm t\bar{s}})$ and $\Gamma({\rm H^+} \to {\rm t\bar{d}})$ in the different ansätze using $M_{\rm H^+} = 250$ GeV and $\psi_D = \theta_D = \frac{\pi}{18}$

Γ (GeV)	FA	DXA	FDXA	DXFA	$\widetilde{\rm X}$A	FNA	NTA
$\Gamma({\rm H^+} \to {\rm c\bar{s}})$	8.06×10^{-4}	6.29×10^{-5}	1.02×10^{-4}	5.57×10^{-5}	5.82×10^{-5}	4.68	6.21×10^{-5}
$\Gamma({\rm H^+} \to {\rm c\bar{b}})$	1.74×10^{-2}	6.45×10^{-5}	1.76×10^{-2}	1.87×10^{-4}	7.35×10^{-5}	8.80×10^{-3}	5.26×10^{-6}
$\Gamma({\rm H^+} \to {\rm t\bar{b}})$	1.32	1.34	1.34	1.34	1.34	1.34	1.34
$\Gamma({\rm H^+} \to {\rm t\bar{s}})$	6.12×10^{-2}	4.57×10^{-4}	7.71×10^{-3}	2.64×10^{-2}	1.38×10^{-3}	2.23×10^{-3}	1.38×10^{-3}
$\Gamma({\rm H^+} \to {\rm t\bar{d}})$	2.69×10^{-3}	2.71×10^{-5}	6.44×10^{-4}	1.41×10^{-3}	6.90×10^{-5}	2.03×10^{-4}	7.24×10^{-5}

type and D-type quarks. In this case, vanishing entries of the ξ^U and ξ^D matrices are obtained; this implies the absence of flavor changing neutral currents involving terms of the form $h^0\bar{\mathrm{u}}\mathrm{c}$, $h^0\bar{\mathrm{c}}\mathrm{t}$, $h^0\bar{\mathrm{d}}\mathrm{b}$ and $h^0\bar{\mathrm{s}}\mathrm{b}$. Finally, a new type of ansatz has been proposed, in which the FCNC involving the top quark have vanished completely, and excellent agreement with the CKM experimental elements is gotten, with the exception of the $|K_{\mathrm{ub}}|$, $|K_{\mathrm{cb}}|$ and $|K_{\mathrm{td}}|$ elements. Our results are shown in Tables 1 and 2.

On the other hand, a discussion of the phenomenology of the two body decays of the lightest Higgs boson, the top quark and the charged Higgs boson using the different mass matrices ansätze in the framework of 2HDM type III is presented in Sect. 4, and our results are shown in Tables 3–5. For the lightest neutral Higgs boson, the decays $h^0 \to \mathrm{b}\bar{\mathrm{b}}$ and $h^0 \to \mathrm{b}\bar{\mathrm{s}}$ are interesting. In the different ansätze considered, the channel $\mathrm{b}\bar{\mathrm{b}}$ can be enhanced with respect to the SM using appropriate values of the Yukawa parameters. The channel $\mathrm{b}\bar{\mathrm{s}}$ would be more important in the Fritzsch ansatz than in the other ansätze. For the charged Higgs decays two options have been explored: a lighter charged Higgs boson and one heavier than the top quark. Taking a lighter charged Higgs boson is a possibility to get a bigger decay width for $H^+ \to \mathrm{c}\bar{\mathrm{b}}$ than the decay width for $H^+ \to \mathrm{c}\bar{\mathrm{s}}$, depending on the ansatz used. In the case of a charged Higgs boson heavier than the top quark, if the Fukuyama–Nishiura ansatz is used, the most important channel would be $H^+ \to \mathrm{c}\bar{\mathrm{s}}$. Also in the case of a charged Higgs boson heavier than the top quark, the channels with a top quark in the final state would be relevant in an eventual search for the charged Higgs boson.

Acknowledgements. We acknowledge R. Diaz for useful discussions. This work has been supported by COLCIENCIAS and the Fundación Banco de la Republica.

Appendix A: CKM mixing matrix

The corresponding standard parametrization of the CKM mixing matrix is [44],

$$K = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}\mathrm{e}^{-\mathrm{i}\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}\mathrm{e}^{\mathrm{i}\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}\mathrm{e}^{\mathrm{i}\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}\mathrm{e}^{\mathrm{i}\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}\mathrm{e}^{\mathrm{i}\delta} & c_{23}c_{13} \end{pmatrix} \tag{A.1}$$

where $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$, with $i, j = 1, 2, 3$, and where δ is the CP violating phase.

Appendix B: Quark masses

In the numerical computations, we used the central values of the following quark masses at the energy scale of the order of m_Z [44]:

$$\begin{aligned} m_{\mathrm{u}}(m_Z) &= 1.64 \pm 0.40\,\mathrm{MeV}\,, \\ m_{\mathrm{c}}(m_Z) &= 620 \pm 30\,\mathrm{MeV}\,, \\ m_{\mathrm{t}}(m_Z) &= 172.7 \pm 2.9\,\mathrm{GeV}\,, \end{aligned} \tag{B.1}$$

$$\begin{aligned} m_{\mathrm{d}}(m_Z) &= 2.92 \pm 0.60\,\mathrm{MeV}\,, \\ m_{\mathrm{s}}(m_Z) &= 55.56 \pm 8.00\,\mathrm{MeV}\,, \\ m_{\mathrm{b}}(m_Z) &= 2.85 \pm 0.18\,\mathrm{GeV}\,. \end{aligned} \tag{B.2}$$

References

1. H. Fritzsch, Phys. Lett. B **70**, 436 (1977)
2. H. Fritzsch, Phys. Lett. B **73**, 317 (1978)
3. H. Fritzsch, Nucl. Phys. B **155**, 189 (1979)
4. H. Fritzsch, J. Plankl, Phys. Lett. B **237**, 451 (1990)
5. H. Fritzsch, Z. Xing, Phys. Lett. B **555**, 63 (2003) [hep-ph/0212195]
6. H. Fritzsch, Prog. Part. Nucl. Phys. **45**, 1 (2000) [hep-ph/9912358]
7. H. Fritzsch, Nucl. Phys. B **556**, 49 (1999) [hep-ph/9904286]
8. H. Fritzsch, Phys. Lett. B **353**, 114 (1995) [hep-ph/9502297]
9. L. Lavoura, Int. J. Mod. Phys. A **9**, 1873 (1994)
10. Y.F. Zhou, J. Phys. G **30**, 783 (2004) [hep-ph/0307240]
11. Y.F. Zhou, hep-ph/0309076
12. G.F. Giudice, Mod. Phys. Lett. A **7**, 2429 (1992) [hep-ph/9204215]
13. T. Fukuyama, H. Nishiura, Proceedings of 1997 Shizuoka Workshop of Masses and Mixings of Quarks and Leptons (1997) pp. 19–21 [hep-ph/9702253]
14. D. Cremades, L.E. Ibanez, F. Marchesano, JHEP **0307**, 038 (2003)
15. D. Cremades, L.E. Ibanez, F. Marchesano, JHEP **0405**, 079 (2004)
16. P. Ramond, R.G. Roberts, G.G. Ross, Nucl. Phys. B **406**, 19 (1993)
17. G.C. Branco, L. Lavoura, F. Mota, Phys. Rev. D **39**, 3443 (1989)
18. S. Dimopoulos, J.L. Hall, S. Raby, OSU, preprint-DOE-ER-01545-567 (1991)
19. H. Georgi, C. Jarlskog, Phys. Lett. B **86**, 297 (1979)
20. K. Matsuda, H. Nishiura, Phys. Rev. D **69**, 053 005 (2004) [hep-ph/0309272]
21. K. Matsuda, T. Fukuyama, H. Nishiura, Phys. Rev. D **61**, 053 001 (2000) [hep-ph/9906433]
22. T.P. Cheng, M. Sher, Phys. Rev. D **35**, 3484 (1987)
23. G.C. Branco, D. Emmanuel-Costa, R. Gonzalez Felipe, Phys. Lett. B **477**, 147 (2000)
24. G.C. Branco, M.N. Rebelo, J. Silva-Marcos, Phys. Lett. B **597**, 155 (2004) [hep-ph/0403016]
25. G.C. Branco, L. Lavoura, J. Silva, *CP* Violation (Clarendon Press, Oxford, 1999)
26. C. Jarlskog, Phys. Rev. Lett. **55**, 1039 (1985)
27. S.N. Gupta, J.M. Johnson, Phys. Rev. D **44**, 2110 (1991)
28. D. Du, Z.Z. Xing, Phys. Rev. D **48**, 2349 (1993)
29. H. Fritzsch, Z.Z. Xing, Phys. Lett. B **353**, 114 (1995)
30. Y. Koide, H. Nishiura, K. Matsuda, T. Kikuchi, T. Fukuyama, Phys. Rev. D **66**, 093 006 (2002) [hep-ph/0209333]

31. N. Haba, K. Yoshioka, Nucl. Phys. B **739**, 254 (2006) [hep-ph/0511108]
32. T. Fukuyama, H. Nishiura, hep-ph/9702253
33. A. Datta, P.J. O'Donnell, Phys. Rev. D **72**, 113 002 (2005) [hep-ph/0508314]
34. K. Kang, S.K. Kang, Phys. Rev. D **56**, 1511 (1997) [hep-ph/9704253]
35. H. Nishiura, K. Matsuda, T. Kikuchi, T. Fukuyama, Phys. Rev. D **65**, 097 301 (2002) [hep-ph/0202189]
36. S. Glashow, S. Weimberg, Phys. Rev. D **15**, 1958 (1977)
37. Y.L. Wu, L. Wolfenstein, Phys. Rev. Lett. **73**, 1762 (1994)
38. W.S. Hou, Phys. Lett. B **296**, 179 (1993)
39. R. Diaz, R. Martinez, J.A. Rodriguez, Phys. Rev. D **63**, 095 007 (2001)
40. R. Diaz, R. Martinez, J.A. Rodriguez, Phys. Rev. D **64**, 033 004 (2001)
41. R. Diaz, R. Martinez, J.A. Rodriguez, Phys. Rev. D **67**, 075 011 (2003)
42. J.F. Gunion, H.E. Haber, Phys. Rev. D **67**, 075 019 (2003) [hep-ph/0207010]
43. J.F. Gunion, H.E. Haber, Phys. Rev. D **72**, 095 002 (2005) [hep-ph/0506227]
44. Particle Data Group, G.-M. Yao et al., J. Phys. G **33**, 1 (2006)
45. L.-L. Chau, W.-Y. Keung, Phys. Rev. Lett. **53**, 1802 (1984)
46. H. Harari, M. Leurer, Phys. Lett. B **181**, 123 (1986)
47. F.J. Botella, L.-L. Chao, Phys. Lett. B **168**, 97 (1986)
48. H. Fusaoka, Y. Koide, Phys. Rev. D **57**, 3986 (1998) [hep-ph/9712201]